INTEGRATED INERTER DESIGN AND APPLICATION TO OPTIMAL VEHICLE SUSPENSION SYSTEM

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ABSTRACT

The formula cars need high tire grip on racing challenge by reducing rolling displacement at corner or double change lands. In this case study, the paper clarifies some issues related to suspension system with inerter to reduce displacement and rolling angle under impact from road disturbance on Formula SAE Car. We propose some new designs, which have an advance for suspension system by improving dynamics. We optimize design of model based on the minimization of cost functions for roll dynamics, by reducing the displacement transfer and the energy consumed by the inerter. Base on a passive suspension model that we carried out quarter-car and half-car model for different parameters which show the benefit of the inerter. The important advantage of the proposed solution is its integration a new mechanism, the inerter, this system can increase advance in development and have effects on the vehicle dynamics in stability vehicle.

KEYWORDS

Inerter, Suspension System, Optimal, Formula SAE, Rolling.

1.INTRODUCTION

Passive, semi-active and active suspension systems have been utilized to improve ride comfort of vehicles and their effectiveness has also been demonstrated [1]. However, it is not easy to improve rolling comfort and dynamics stability with passive suspension systems [2]. To achieve it, several control methods have been proposed, but most of them relate the active suspension [3]. In this case study, the passive suspension presented as the simple system that can be improve rolling stability depend on the sensitivity of the system parameters that take and consider to be introduced [4].

To improve rolling stability, this study proposes a design method passive suspension system taking with new component element named "inerter" into consideration the both sensitive of the sprung and un-sprung mass vehicle behaviour when have road disturbance [5]. A method that can improve both the rolling and the displacement of vehicle body is proposed by optimizing the modal parameters of suspension and tire. Furthermore, the optimization is scheduled in the time domain to attain the optimal values of parameters during impact period. The dynamics of road disturbance is assumed to make for initial conditions. In order to verify the effectiveness of the proposed method, a half-car model that has variable stiffness, damping and inerter suspension system is constructed and the numerical simulations are carried out.

For modelling of an inerter, it was defined to be a mechanical two-terminal, one-port device with the property that the equal and opposite force applied at the nodes is proportional to the relative acceleration between the nodes through a rack, pinion, and gears Figure 1. To approximately model the dynamics of the device, let r_1 be the radius of the rack pinion, r_2 the radius of the gear

wheel, r_3 the radius of the flywheel pinion, γ the radius of gyration of the flywheel, m the mass of the flywheel [6].
That that the following relation holds:

$$F = b(\dot{v}_2 - \dot{v}_1) \quad (1)$$

The constant of proportionality b is called the inertance and has units of kilograms:

$b = m\alpha_1^2\alpha_2^2$
Where $\alpha_1 = \gamma/r_3$ and $\alpha_2 = r_2/r_1$.
It stored energy equal to $(1/2)b(v_2 - v_1)^2$ (2)

Let us focus attention first on the familiar two-terminal modeling elements: spring, damper and inerter. Each is an ideal modeling element, with a mathematical definition. It is useful to discuss on mechanical networks, which give some hint toward the inerter idea, in order to highlight the new passive suspension system relate to rolling problems.

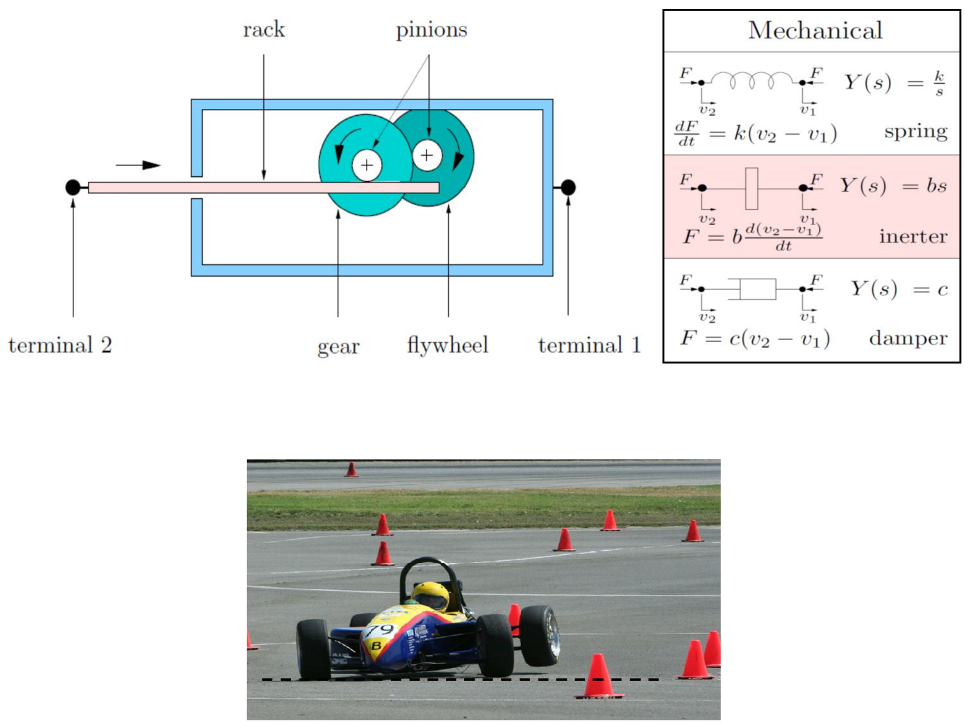
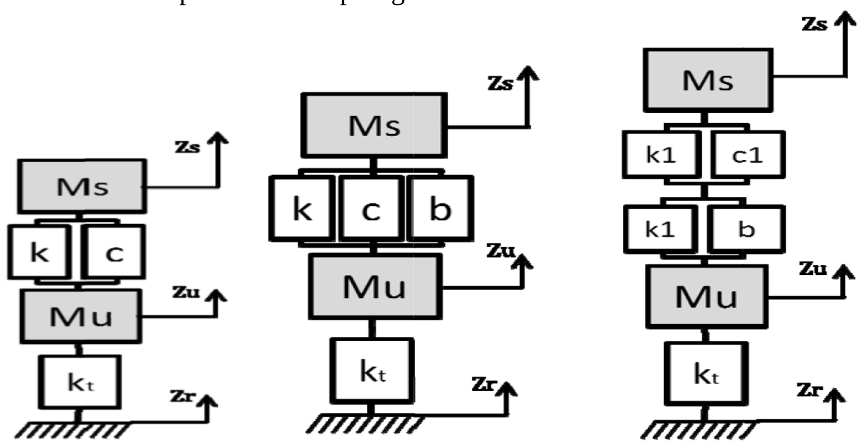


Figure1.The theory structural inerter element correspondence for the mechanical network and rolling problems on Formula SAE car [7].

2. MATHEMATICAL MODEL

2.1. Quarter-car Model

Base on the conventional quarter- car model, we design two structures are called quarter-car suspension parallel and series structure model (Figure 2). This model will change from normal passive suspension to new suspension with stiffness, damping and inerter in parallel or series. We study about the vertical displacement of sprung mass in some kinds of simulations.



(a) (b) (c)

Figure 2: The base, the parallel and the series quarter-car model in respectively.

For the quarter-car model, the suspension strut provides an equal and opposite force on the sprung and un-sprung masses by means of the positive real admittance function which relates the suspension force to the strut velocity through spring, damper and inerter. We define them as the following equations.

Base quarter-car model dynamics equation in time-domain:

$$M_s\ddot{Z}_s(t) = F_k(t) + F_c(t)$$
$$M_u\ddot{Z}_u(t) = F_{kt}(t) - (F_k(t) + F_c(t)) \qquad (3)$$

Where:

$$F_k(t) = k(Z_u(t) - Z_s(t))$$
$$F_c(t) = c(\dot{Z}_u(t) - \dot{Z}_s(t)) \qquad (4)$$
$$F_{kt}(t) = k_t(Z_r(t) - Z_u(t))$$

State-space representation:

$$\begin{bmatrix} \ddot{Z}_s \\ \dot{Z}_s \\ \ddot{Z}_u \\ \dot{Z}_u \end{bmatrix} = \begin{bmatrix} \frac{-c}{M_s} & \frac{-k}{M_s} & \frac{c}{M_s} & \frac{k}{M_s} \\ 1 & 0 & 0 & 0 \\ \frac{c}{M_u} & \frac{k}{M_u} & \frac{-c}{M_u} & \frac{-k-k_t}{M_u} \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{Z}_s \\ Z_s \\ \dot{Z}_u \\ Z_u \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{k_t}{M_u} \\ 0 \end{bmatrix} Z_r \qquad (5)$$

2.2. Suspension Design

We summarize the approach of the suspension design problem was formulated as an optimal modal parameter to improve vertical displacement and rolling angle. The solution of the optimization problem made use structure of new quarter-car model that improve from traditional passive suspension system in adding inerter elements. In some previous researching, the good and simple structure was able to come up with new network topologies involving inerter. To solving these problems, we use Laplace transform function as the ways to represent for suspension system in cases study by Q(s).

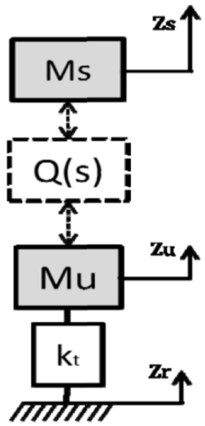


Figure 3. The quarter-car model with suspension function represented in Laplace transformed.

The equations of motion in the Laplace transformed domains are:

$$M_s s^2 \widehat{Z_s} = -sQ(s)(\widehat{Z_s} - \widehat{Z_u})$$
$$M_u s^2 \widehat{Z_u} = sQ(s)\left(\widehat{Z_s} - \widehat{Z_u}\right) + k_t\left(\widehat{Z_r} - \widehat{Z_u}\right) \qquad (6)$$

We can compute the relevant transfer functions as follows:

The transfer functions from the road disturbance z_r to the displacement of the sprung mass z_s:

$$T_{Z_r \to Z_s} = \frac{k_t Q(s)}{M_s s(M_u s^2 + k_t) + ((M_u + M_s)s^2 + k_t)Q(s)} \qquad (7)$$

The transfer functions from the road disturbance z_r to the acceleration of the sprung mass $\ddot{Z}_s$:

$$T_{Z_r \to \ddot{Z}_s} = s^2 T_{Z_r \to Z_s} = \frac{s^2 k_t Q(s)}{M_s s(M_u s^2 + k_t) + ((M_u + M_s)s^2 + k_t)Q(s)} \qquad (8)$$

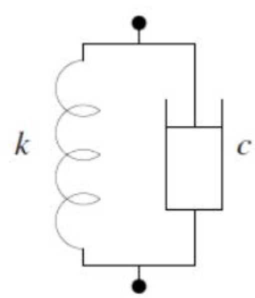


Figure 4. The conventional parallel spring-damper arrangement

The conventional suspension function represented in Laplace transformed:

$$Q(s) = Y_k + Y_c = \frac{k}{s} + c \qquad (9)$$

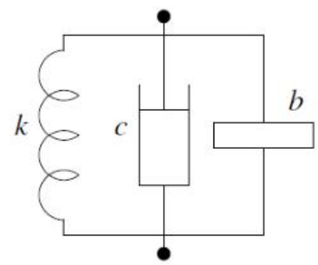


Figure 5.The parallel spring-damper augmented by an inerter in parallel.

The parallel suspension function represented in Laplace transformed:

$$Q(s) = Y_k + Y_c + Y_b = \frac{k}{s} + c + bs \quad (10)$$

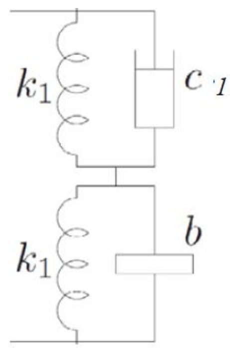


Figure 6.The parallel spring-damper series with the parallel spring-inerter.

The series suspension function represented in Laplace transformed:

$$Q(s) = \left(\frac{1}{Y_c + Y_{k_1}} + \frac{1}{Y_b + Y_{k_2}}\right)^{-1} = \left(\frac{1}{c + \frac{k_1}{s}} + \frac{1}{bs + \frac{k_2}{s}}\right)^{-1} \quad (11)$$

3. Quarter-car Model Analysis

3.1. Specification of Quarter-car Model

For the evaluation of the displacement of the sprung mass, we use a hump road profile and there is no load disturbances applied on the sprung mass. Base on previous study, we have modal parameters for passive suspension system as:

Table1. The specification of Formula SAE quarter-car model.

Symbols	Parameters	Values
M_s	Mass of body	63 kg
M_u	Mass of tire	12 kg
k	Stiffness coefficient	24000 N/m
k_1	Stiffness coefficient	48000 N/m
c	Damping coefficient	1200 Ns/m
c_1	Damping coefficient	2400 Ns/m
b	Mass of inertance	20 kg
k_t	Stiffness coefficient of tire	70000 N/m
H_0	Road disturbance hump	0.05 m

We have looked at suspension and steering systems in regard to vehicle vibration characteristics affecting vehicle-mounted equipment. Figure 7 shows related vehicle vibration and noise classified according to source input and source frequency.

Just as in the suspension system, most vibration affecting the steering system does not act directly on the steering system, but is amplified in the tires and suspension system. Other vibration phenomena include power steering vibration, kickback due to uneven road disturbance, and dynamic unbalance of rotating parts such as the wheels and the braking system.

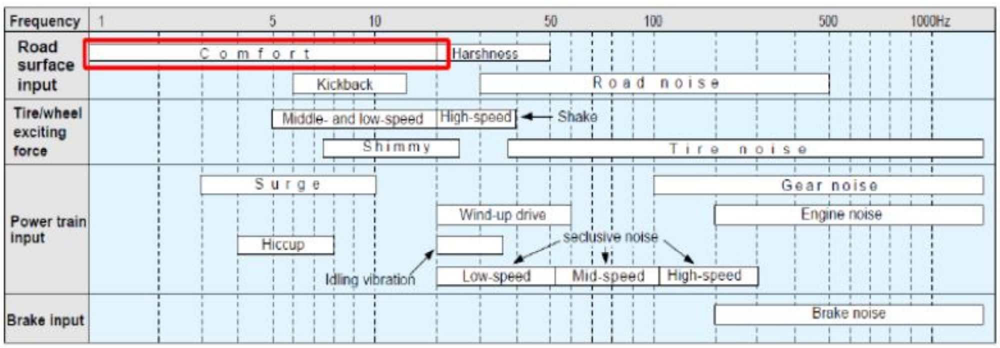


Figure 7. Vibration sources and input to the suspension [8].

Low-speed shimmy is generated by self-induced vibration accumulating within the steering system, and occurs at the low vehicle speeds of 20 to 60 kilometres per hour. Displacement is large, and the shimmy has a greater tendency to occur in worn tires or tires with low air pressure. High-speed shimmy is mainly caused by static or dynamic unbalance of the tires on the wheels. Other causes include disk wheel eccentricity, the wheel not being vertically plumb, and non-uniformity of the tires. Unbalance of the tires on the wheels causes peak vibration in the vicinity of 10 Hz to 30 Hz. Displacement is small, but with worn tires or low air pressure in the tires, displacement becomes large.

Kickback occurs when driving on a bad road surface causes both vertical and horizontal vibration to the drive tires. That vibration is transmitted to the tie rods, causing the steering wheel to turn suddenly. This vibration occurs in the drive wheels on front-wheel drive cars, giving a greater tendency for longitudinal changes in force than with rear-wheel-drive cars.

Tire vibration factors seen from the characteristics of static springs; include vertical springs, lateral springs, longitudinal springs, and torsion springs. Longitudinal stiffness is strongly related to actual vibration, and becomes harder roughly in proportion to the increase in air pressure. Natural tire frequency results from the fact that an elastic body filled with air has an individual vibration mode.

In this study, we focus to analyze the effect from road disturbance to kickback and comfort via displacement and acceleration of sprung mass. The frequency domain to estimate from 1-20 Hz, it is the main reason that we not mention about other disturbances.

3.2. Mathematical Model Analysis

The present numerical discussion consists in analyzing the previous model properties on a numerical example. The model used in the following simulations is the simple LTI passive one, as given in definition with the Formula SAE car parameters given in Table 1. Since the main objective is to analyze passive suspensions, suspensions with inerter employed and changeable, the analysis carried in this section is cantered on the behaviour of the passive system for varying inertance b values.

Figure 8 illustrates the pole location of the passive quarter-car model for different models, with b values 20 kg. First, according to this figure, it is notable that whatever the frequency smaller than 5 Hz, parallel model is better than base model while the series model is unstable. The quarter-car system remains stable with parallel structure. However, from 5 Hz to 12 Hz, the parallel model is not good as base model, and the series model is remaining unstable.

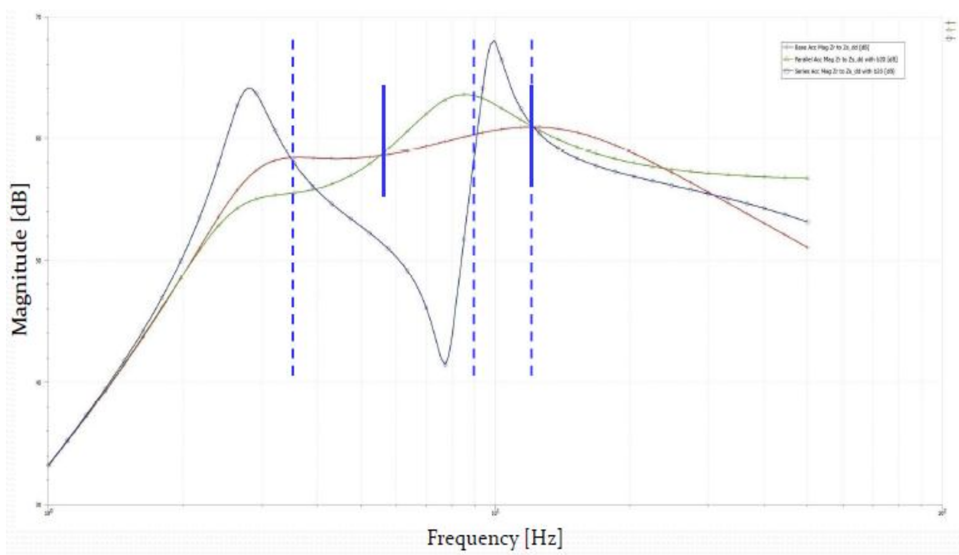


Figure 8. Base, Parallel and Series model in Frequency response of acceleration from road disturbance Z_r to sprung-mass Z_{s_dd}

Then in parallel structure, by increasing the b value, the acceleration response magnitude poles module reduces. More specifically see in Figure 9.

When b = 10 kg, the poles are located on the imaginary axis and start to reduce in 1-5 Hz.

When b→50 kg, the poles tend to be located is continuous reduce in this frequency.
From the figure, it is interesting to note the following points:

On Figure 9, as expected and accordingly to the pole location analysis provided with Figure 8, increase the inertance b value reduce the resonance peaks, leading to an oscillatory behaviour.

But, in other frequency phase, the system is not good.
On Figure 10, the system is unstable with series model. As previously explained, some of these reason when we change b value, we cannot verify the behaviour of model. Indeed on Figure 10,

where analysis is shown for varying b value and fixed stiffness values, damper value, from 1 to 20 Hz, the system seem to be not good at all.

Finally, for improve suspension system in oscillation, we suggest using parallel structure to optimize quarter-car and then is half-car model respectively.

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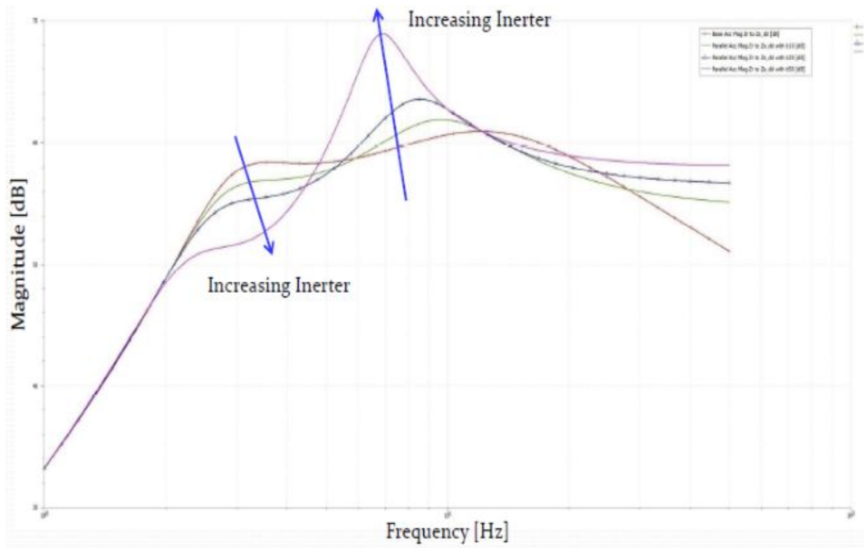

Figure 9. Comparison Base and Parallel model in Frequency response from Z_r to Z_{s_dd} with the change of b_value.

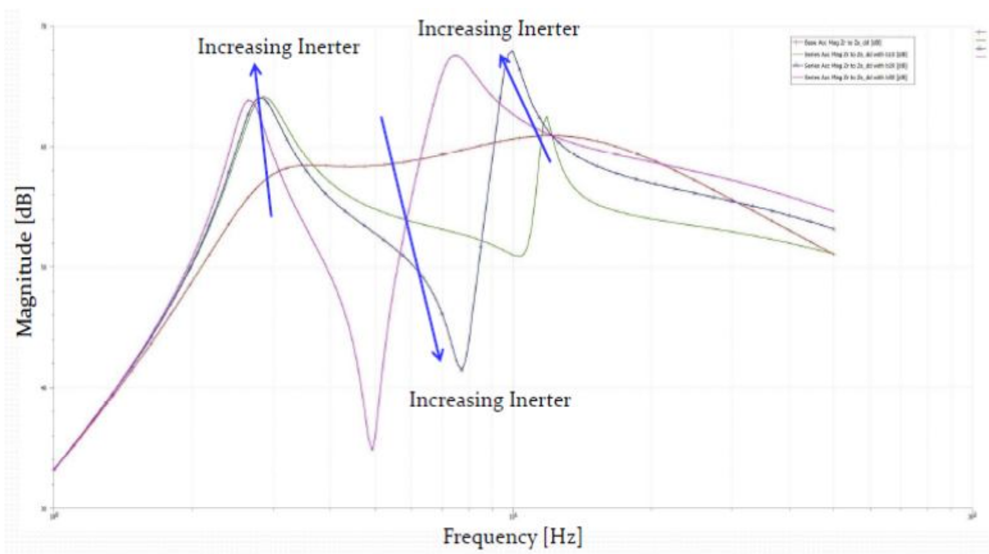

Figure 10. Comparison Base and Series model in Frequency response from Z_r to Z_{s_dd} with the change of b_value.

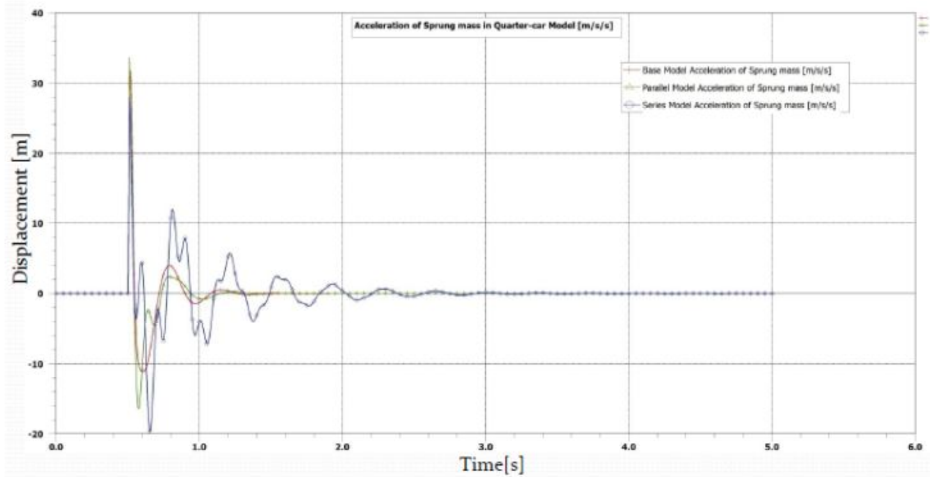

Figure 13. Sprung-mass acceleration results of Base, Parallel and Series model in time-domain.

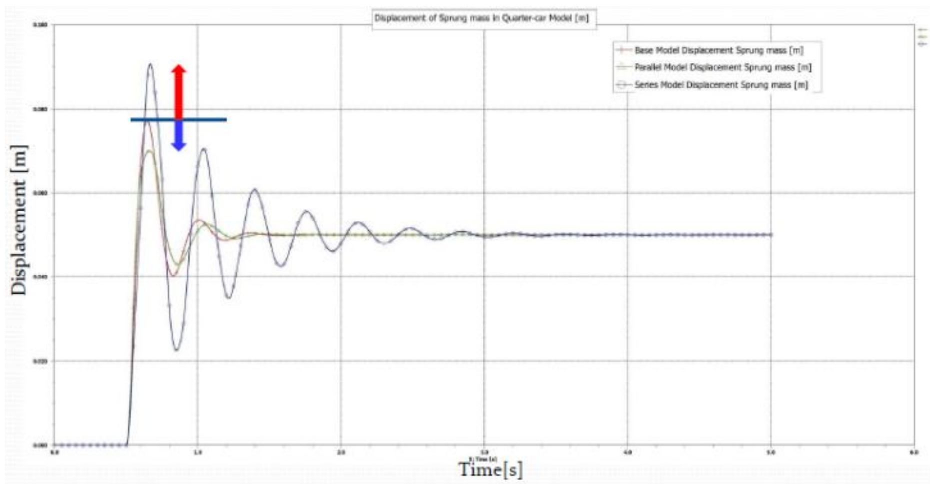

Figure 14. Sprung-mass displacement results of Base, Parallel and Series model in time-domain.

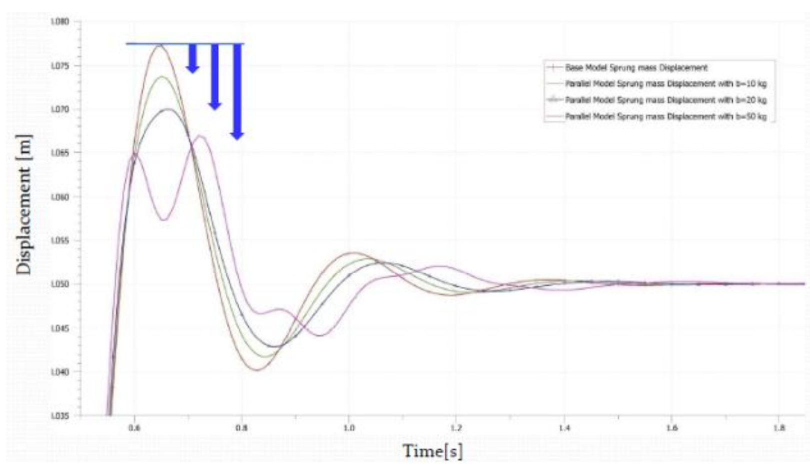

Figure 15. Sprung-mass displacement results of Base Vs Parallel model with change b_value.

The Figures show that the inerter has an advantage in frequency from 2 Hz to 5 Hz, and disadvantage from 6 Hz to 12 Hz with parallel suspension system. The parallel suspension model is more stable than the series suspension model. Frequency response function will be the basis of design suspension system with some points above.

4.OPTIMIZATION QUARTER-CAR MODEL

4.1. Optimization Quarter-car Model Parameters

To optimize displacement, in this section we will focus on a single aspect of performance is related to the dynamics problem. We used the structure parameter which represented the following design variables: k, c, b, and k_t, with lower and upper boundaries showed below.

Table 2. The boundary of design variables.

Lower boundary	Parameter	Upper boundary	Unit
20000	k	28000	N/m
1000	c	1400	Ns/m
10	b	30	kg
60000	k_t	80000	N/m

The objective function is represented for the maximum value of displacement of sprung-mass as:

$$\mathrm{Max}Z_s = f(k, c, b, k_t) \rightarrow \mathrm{Min} \tag{12}$$

Under the synthetic assumption, we can figure out this peak-point of displacement equation:

$$\mathrm{Max}Z_s = 0.052 + 1.114(e-7)k - 6.853(e-12)k^2 - 1.086(e-05)c + 4.597(e-09)c^2 + 2.413(e-07)b - 1.049(e-12)b^2 + 7.957(e-04)k_t - 6.993(e-06)k_t^2 \tag{13}$$

The constrained functions were represented by the tire displacement:

$$\mathrm{Min}\{\mathrm{Max}\, Z_u\} \leq Z_u(k, c, b, k_t) \leq \mathrm{Max}\{\mathrm{Max}\, Z_u\} \tag{14}$$

To calculating Max Z_u, we used Orthogonal Arrays (OA) of Design of Experimental with L_{27} (3^{13}) then, we have:

$$\mathrm{Min}\{\mathrm{Max}\, Z_u\} = 0.0618\ (m) \tag{15}$$

$$\mathrm{Max}\{\mathrm{Max}\, Z_u\} = 0.0756\ (m) \tag{16}$$

We introduce the approximate optimization method, Sequential Quadratic Programming (SQP) with Response Surface Method (RSM). We made RSM using the OA to optimize modal parameters of the passive suspension system with inerter. The new modal parameters are represented for optimization suspension system (opt_model) and they are called: k_opt, c_opt, b_opt and k_t_opt parameters.

In order to verify the effectiveness of the proposed optimal method, the numerical simulations were carried out. The vertical displacement optimization results were measured over various fixed structure suspensions. The optimization was performed for k_opt, c_opt, b_opt and k_t_opt ranging from boundary conditions. The modal parameter results were obtained by fixed-structure is presented in Table.3.

Table 3. The comparative modal parameters for quarter-car model.

Parameter	**k**	**c**	**b**	**k_t**
old_model	24000	1200		70000
new_model	24000	1200	20	70000
opt_model	**20727**	**1363**	**28**	**78182**
Unit	N/m	Ns/m	kg	N/m

4.2.Results and Discussions of Quarter-car Model

The results were confirmed that the suspension system with inerter effects to the displacement of sprung-mass, the time histories of the displacement are shown in Figure 16, respectively. From the results, it was verified that the displacement is reduced by comparison between old and new models.

The optimization result is presented as circle-symbol curve suggesting that the structure of the suspension optimize from the stiffness, damper and inerter. An encouraging feature of the optimization algorithm that it allows the change in the structure of suspension parameters varies in order to obtain the minimum vertical displacement values.

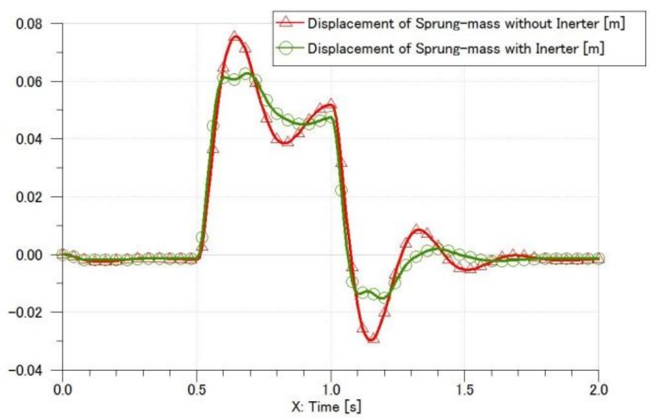

Figure 16. The displacement of sprung-mass on quarter-car model.

Comparing among old-model and opt-model we found that correspondences between these peaks of displacement variables of sprung-mass is summarized below. All results verified with new suspension system could be accomplished that the body displacement dynamics can be reduced with employment inerter component.

Table 4. Comparison the peaks of vertical displacement results for quarter-car model

Displacement peak	**Result (m)**	**Improvement**
Z_s_old	0.0752	0%
Z_s_opt	**0.0629**	**16.36%**

5. OPTIMIZATION HALF-CAR MODEL

5.1. Specification of Half-car Model

We summarize the approach of the suspension design problem was formulated as an optimal modal parameter to improve rolling angle through displacement variables. The solution of the optimization problem uses structure of new half-car model that is improved from traditional passive suspension system in adding inerter elements. In this paper, we shall apply the training parameterization method to the half-car model.

In this part, we made simulations creating mathematical model with inerter in the linear half-car model. The vehicle model has an input road disturbance to right sides, and the main outputs are rolling angle (φ) and sprung mass displacement (Z_2). Several subsystems of model performance are considered such as sprung mass, un-sprung mass, suspension and tire (Figure 18). To optimizing suspension problems, we used these simulation results as the initial values that results represent for the picks of rolling angle Maxφ and sprung mass displacement MaxZ_2.

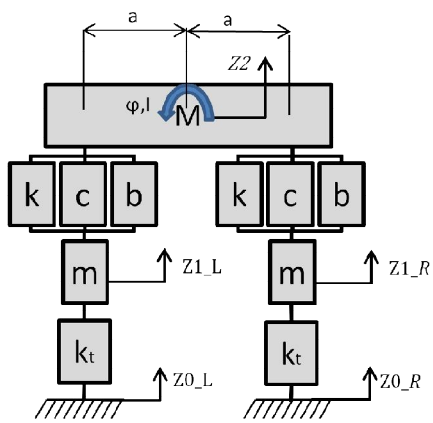

Figure 17. The half-car model.

We shall apply the training parameterization method to the half-car model. The half-car model represented in Figure 17 is the simple model to consider for suspension design. It consists of the sprung mass M, the un-sprung mass m and the tire model represented by stiffness k_t. The suspension strut provides an equal and opposite force on the sprung and un-sprung masses by means of the positive-real admittance function which relates the suspension force to the strut velocity through spring, damper and inerter.

For the half-car model, we define it as the following equations. The module of sprung mass body was represented by these equations:

$$\begin{aligned} &M\ddot{z}_2(t) = (F_{kL}(t) + F_{cL}(t) + F_{bL}(t)) + (F_{kR}(t) + F_{cR}(t) + F_{bR}(t)) \\ &I\ddot{\varphi}(t) = a\left(F_{kL}(t) + F_{cL}(t) + F_{bL}(t)\right) - a\left(F_{kR}(t) + F_{cR}(t) + F_{bR}(t)\right) \\ &I = Ma^2/3 \end{aligned} \quad (17)$$

The module of suspension systems were represented as:

$$\begin{aligned} &F_{kL}(t) = k(Z_{1L}(t) - (Z_2(t) - a\varphi(t))) \\ &F_{cL}(t) = c(\dot{Z}_{1L}(t) - (\dot{Z}_2(t) - a\dot{\varphi}(t))) \\ &F_{bL}(t) = b(\ddot{Z}_{1L}(t) - (\ddot{Z}_2(t) - a\ddot{\varphi}(t))) \end{aligned} \quad (18)$$

$$F_{kR}(t) = k(Z_{1R}(t) - (Z_2(t) + a\varphi(t)))$$
$$F_{cR}(t) = c(\dot{Z}_{1R}(t) - (\dot{Z}_2(t) + a\dot{\varphi}(t)))$$
$$F_{bR}(t) = b(\ddot{Z}_{1R}(t) - (\ddot{Z}_2(t) + a\ddot{\varphi}(t)))$$

The un-sprung mass module as:

$$m\ddot{Z}_{1L}(t) = F_{ktL}(t) - (F_{kL}(t) + F_{cL}(t) + F_{bL}(t))$$
$$m\ddot{Z}_{1R}(t) = F_{ktR}(t) - (F_{kR}(t) + F_{cR}(t) + F_{bR}(t)) \quad (19)$$

The tire module as:

$$F_{ktL}(t) = k_t(Z_{0L}(t) - Z_{1L}(t))$$
$$F_{ktR}(t) = k_t(Z_{0R}(t) - Z_{1R}(t)) \quad (20)$$

For the evaluation of the rolling angle we use a hump road profile. The hump has height H_{0_R} (on the right side) and flat on the left side. The hump initially appears after 1 second when run simulation then the right wheel impact while the left wheel has no disturbance. There is no load disturbances applied on the sprung mass.

Base on previous study, we have modal parameters for passive suspension system as:

Table 5. The specification of Formula SAE half-car model.

Symbols	**Parameters**	**Values**
M	Mass of body	126 kg
m	Mass of tire	12 kg
I	Roll moment of inertia	15.1 kgm2
k	Stiffness coefficient	24000 N/m
c	Damping coefficient	1200 Ns/m
b	Mass of inertance	20 kg
k_t	Stiffness coefficient of tire	70000 N/m
a	Half length of track	0.6 m
H_{0_R}	Road disturbance hump	0.1 m

In this part, we made simulations creating two models called: old_model on basic suspension system and new_model with inerter. The methodology used of a linear half-car model which is constructed in the simulation toolboxes. The vehicle model has an input road disturbance right sides, the main output is rolling angle (φ) and sprung mass displacement (Z_2). Several subsystems of model performance are considered such as sprung mass, un-sprung mass, suspension and tire showed in Figure 18. For each aspect of performance we will propose time-domain performance measures that are evaluated after a simulation run. To optimizing suspension problems, we used these simulation results as the initial values. The results represent for the peaks of rolling angle Maxφ and sprung mass vertical displacement $MaxZ_2$.

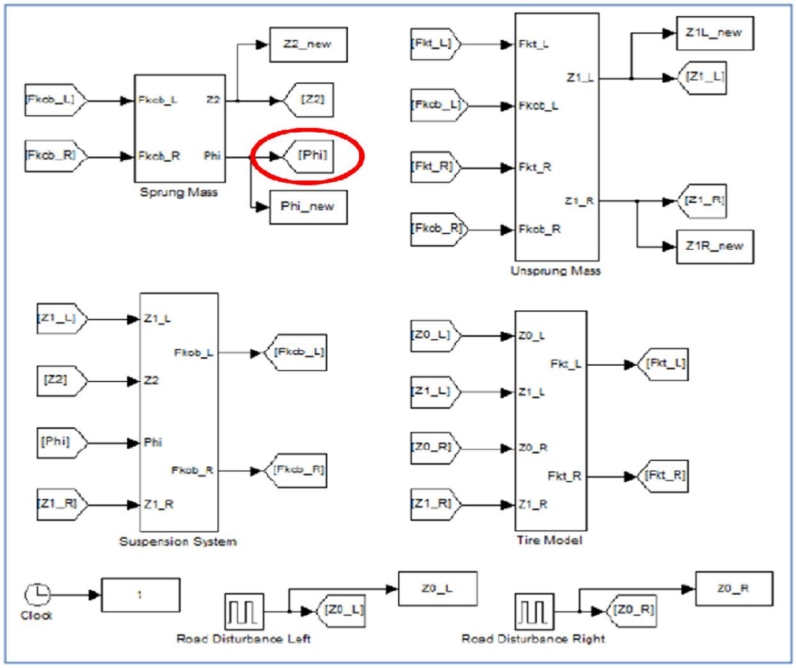

Figure 18. The simulation of half-car model to calculate rolling and displacement.

3.2. Optimization Half-car Model Parameters

For the optimization of rolling, in this section we will focus on a single aspect of performance is related to the rolling problem. We used the structure parameter which represented the following design variables: k, c, b, and k_t, with lower and upper boundaries showed below.

Table 6. The boundary of design variables.

Lower boundary	Parameter	Upper boundary	Unit
20000	k	28000	N/m
1000	c	1400	Ns/m
10	b	30	kg
60000	k_t	80000	N/m

The objective function is represented for the rolling angle modes as:

$$\varphi = \varphi(k, c, b, k_t) \rightarrow \text{Min} \qquad (21)$$

Under the synthetic assumption of basic passive half-car model, we can figure out this rolling equation:

$$\varphi = 0.159 - 5.236(e-07)k + 2.118(e-11)k^2 - 1.35(e-5)c + 1.138(e-08)c^2 - 1.136(e-06)k_t + 6.555(e-12)k_t^2 - 7.75(e-4)b + 2.838(e-05)b^2 \qquad (22)$$

Constrain functions were represented by the body displacement modes under boundaries above. In this case, the results show:

$$\text{Min}\{\text{Max } Z_2\} \leq Z_2(k, c, b, k_t) \leq \text{Max}\{\text{Max } Z_2\} \qquad (23)$$

To calculating Max Z_2, we used Orthogonal Arrays (OA) of Design of Experimental with L_{27} (3^{13}) then:

$$\text{Min}\{\text{Max } Z_2\} = 0.0641 \text{ (m)}$$

$$\text{Max}\{\text{Max } Z_2\} = 0.0755 \text{ (m)} \quad (24)$$

With the same optimal methods, we introduce the approximate optimization method, Sequential Quadratic Programming (SQP) with Response Surface Method (RSM). We made RSM using the OA to optimize modal parameters for half-car model. The new modal parameters are represented for optimization suspension system (opt_model) and they called: k_opt, c_opt, b_opt and k_t_opt.

Table 7. The comparative modal parameters for half-car model.

Parameter	**k**	**c**	**b**	**k_t**
old_model	24000	1200	X	70000
new_model	24000	1200	20	70000
opt_model	**20727**	**1036**	**16.63**	**78182**
Unit	N/m	Ns/m	kg	N/m

For the half-car model, the results were confirmed that the inerter is added to affect the roll angle, the time histories of the body roll angles are shown in Figure 19, respectively. From the results, it was verified that the rolling angle is reduced by comparison among each models.

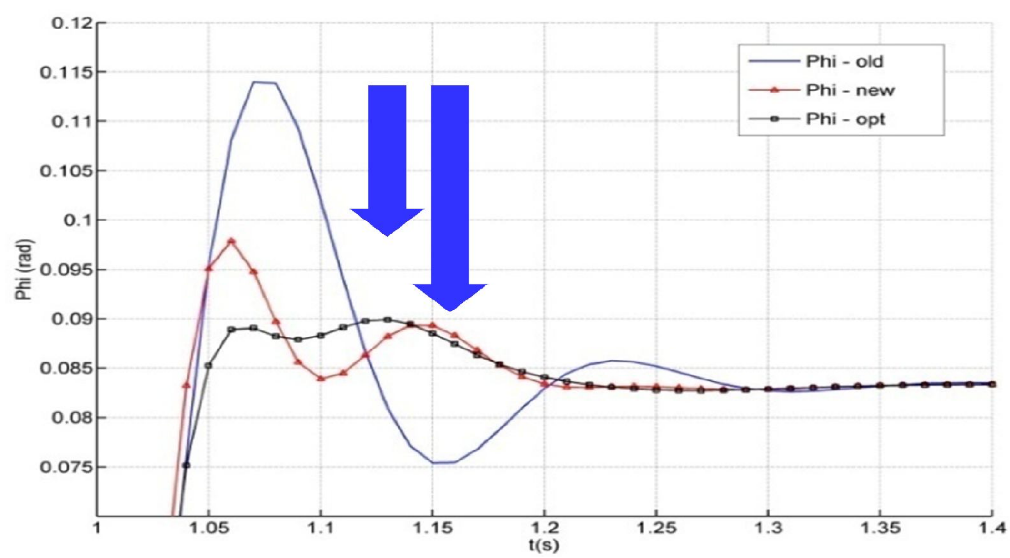

Figure 19. Results of optimization rolling angle for half-car models.

The optimization result is presented as circle-symbol curve suggesting that the structure used the optimal parameters of stiffness, damper and inerter. Comparing among old-model, new-model and opt-model we found that the rolling angle is reduced, as summarized below.

Table 8. The comparison rolling peak results

Rolling peak	**Result (rad)**	**Improvement**
φ_old	0.114	0%

φ_new	0.098	14.03%
φ_opt	**0.090**	**21.05%**

The table shows that with inerter components are employed to passive suspension system as a parallel structure, the rolling angle will be reduced. We just use for a simple and fix inerter parameter b, the rolling value reduce 14 percent, while it will be reduced more than 21 percent if we optimal suspension parameter. In conclusion, the inerter component has an advanced effect to the suspension system in general and in passive suspension system in particular.

6.CONCLUSIONS

This paper has described the background and application of a new element called inerter through the suspension synthesis in Formula SAE car. The passive suspension was considered that is possible application of the inerter. The parallel suspension system is more stable than series, and can be designed to improve vehicle dynamics. The results showed that suspension with inerter was not only have better displacement but also have smaller rolling angle of sprung mass body on quarter-car and half -car model.

It was showed that conventional spring and damper always resulted in very normal vibration behaviour, but the use of inerter can reduce the oscillation. In this studying, an optimal design to achieve variables stiffness, damping and inerter in suspension system achieved better results for rolling with road disturbance. These simulations confirmed that ride comfort in the same frequency domain with basic suspension system while the body car rolling was improved.

Furthermore, base on these advanced optimization results; we verify that the suspension controlled under simple conditions to apply on normal car. We will integrate other types of inerter which can be controlled, and apply on suspension system for large dynamic stability. We should construct some physical modeling for suspension systems then it will be validated with mathematical modeling. We need integrate controllable inerter mechanism to optimizing suspension system for other dynamics stability focus to interactive left side and right side, also study with other dynamic attitudes on full-car model.

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