GLOBAL CHAOS SYNCHRONIZATION OF PAN AND LÜ CHAOTIC SYSTEMS VIA ADAPTIVE CONTROL

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ABSTRACT

This paper deploys adaptive control method to derive new results for the global chaos synchronization of identical Pan systems (2010), identical Lü systems (2002), and non-identical Pan and Lü chaotic systems. Adaptive control method is deployed in this paper for the general case when the system parameters are unknown. Sufficient conditions for global chaos synchronization of identical Pan systems, identical Lü systems and non-identical Lü and Pan systems are derived by deploying adaptive control theory and Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the adaptive control method is suitable for achieving the global chaos synchronization of the chaotic systems discussed in this paper. Numerical simulations are shown to illustrate the adaptive synchronization schemes derived in this paper for the Pan and Lü chaotic systems.

KEYWORDS

Adaptive Control, Chaotic Systems, Chaos Synchronization, Pan System, Lü System.

1. INTRODUCTION

Chaos is an interesting nonlinear phenomenon and has been extensively studied in the last two decades [1-40]. The first chaotic system was discovered by Lorenz [1] in 1963, when he was studying weather patterns.

Chaotic systems are highly sensitive to initial conditions and the sensitive nature of chaotic systems is called as the *butterfly effect* [2]. Chaos theory has been applied in many scientific disciplines such as Mathematics, Computer Science, Microbiology, Biology, Ecology, Economics, Population Dynamics and Robotics.

In 1990, Pecora and Carroll [3] deployed control techniques to synchronize two identical chaotic systems and showed that it was possible for some chaotic systems to be completely synchronized. From then on, chaos synchronization has been widely explored in a variety of fields including physical systems [4,5], chemical systems [6], ecological systems [7], secure communications [8-10], etc.

In most of the chaos synchronization approaches, the *master-slave* or *drive-response* formalism is used. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of the synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically. The seminal work on chaos synchronization was published by Pecora and Carroll in 1990 [3].

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In the last few decades, a variety of impressive approaches have been proposed for the global chaos synchronization of chaotic systems such as the OGY method [11], active control method [12-16], adaptive control method [17-22], sampled-data feedback synchronization method [23], time-delay feedback method [24], backstepping method [25-26], sliding mode control method [27-32], etc.

In this paper, we investigate the global chaos synchronization of uncertain chaotic systems, *viz.* identical Pan systems ([33], 2010), identical Lü systems ([34], 2002) and non-identical Pan and Lü systems. Pan system (Pan, Xu and Zhou, 2010) and Lü system (Lü and Chen, 2002) are important paradigms of three-dimensional chaotic systems. In our adaptive controller design, we consider the general case when the parameters of the chaotic systems are unknown.

This paper is organized as follows. In Section 2, we provide a description of the chaotic systems addressed in this paper, *viz.* Pan system (2010) and Lü system (2002). In Section 3, we discuss the adaptive synchronization of identical Pan systems. In Section 4, we discuss the adaptive synchronization of identical Lü systems. In Section 5, we discuss the adaptive synchronization of non-identical Pan and Lü systems. In Section 6, we summarize the main results obtained in this paper.

2. SYSTEMS DESCRIPTION

The Pan system ([33], 2010) is described by the dynamics

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - x_1) \\ \dot{x}_2 &= \gamma x_1 - x_1 x_3 \\ \dot{x}_3 &= -\beta x_3 + x_1 x_2 \end{aligned} \tag{1}$$

where x_1, x_2, x_3 are the states and α, β, γ are positive, constant parameters of the system.

The Pan system (1) is chaotic when the parameter values are taken as

$$\alpha = 10, \ \beta = 8/3 \ \text{ and } \ \gamma = 16$$

The state orbits of the Pan chaotic system (1) are shown in Figure 1.

The Lü system ([34], 2002) is described by the dynamics

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= c x_2 - x_1 x_3 \\ \dot{x}_3 &= -b x_3 + x_1 x_2 \end{aligned} \tag{2}$$

where x_1, x_2, x_3 are the states and a, b, c are positive, constant parameters of the system.

The Lü system (2) is chaotic when the parameter values are taken as

$$a = 36, \ b = 3 \ \text{ and } \ c = 20$$

The state orbits of the Lü chaotic system (2) are shown in Figure 2.

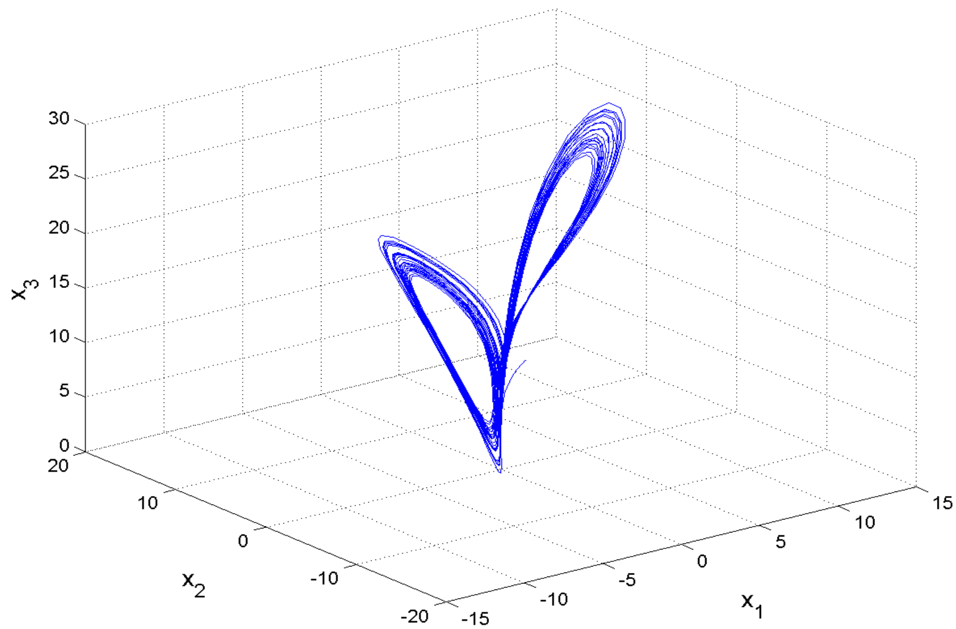

Figure 1. State Orbits of the Pan System

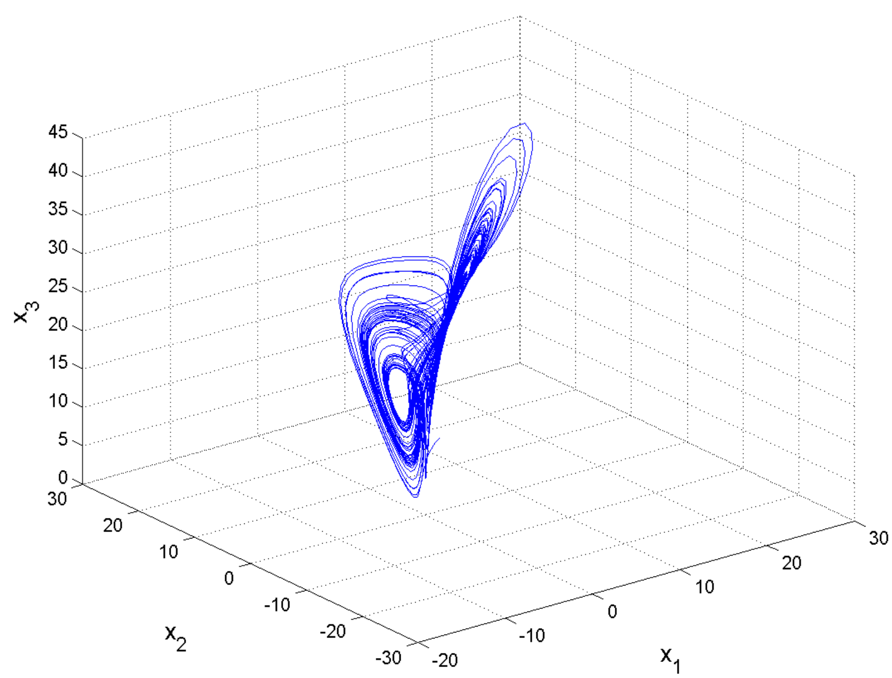

Figure 2. State Orbits of the Lü System

3. ADAPTIVE SYNCHRONIZATION OF IDENTICAL PAN SYSTEMS

3.1 Theoretical Results

In this section, we deploy adaptive control to achieve new results for the global chaos synchronization of identical hyperchaotic Pan systems ([33], 2010), where the parameters of the master and slave systems are unknown.

As the master system, we consider the Pan dynamics described by

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - x_1) \\ \dot{x}_2 &= \gamma x_1 - x_1 x_3 \\ \dot{x}_3 &= -\beta x_3 + x_1 x_2 \end{aligned} \tag{3}$$

where x_1, x_2, x_3 are the states and α, β, γ are unknown, real ,constant parameters of the system.

As the slave system, we consider the controlled Pan dynamics described by

$$\begin{aligned} \dot{y}_1 &= \alpha(y_2 - y_1) + u_1 \\ \dot{y}_2 &= \gamma y_1 - y_1 y_3 + u_2 \\ \dot{y}_3 &= -\beta y_3 + y_1 y_2 + u_3 \end{aligned} \tag{4}$$

where y_1, y_2, y_3 are the states and u_1, u_2, u_3 are the nonlinear controllers to be designed.

The chaos synchronization error is defined by

$$\begin{aligned} e_1 &= y_1 - x_1 \\ e_2 &= y_2 - x_2 \\ e_3 &= y_3 - x_3 \end{aligned} \tag{5}$$

The error dynamics is easily obtained as

$$\begin{aligned} \dot{e}_1 &= \alpha(e_2 - e_1) + u_1 \\ \dot{e}_2 &= \gamma e_1 - y_1 y_3 + x_1 x_3 + u_2 \\ \dot{e}_3 &= -\beta e_3 + y_1 y_2 - x_1 x_2 + u_3 \end{aligned} \tag{6}$$

Let us now define the adaptive control functions

$$\begin{aligned} u_1(t) &= -\hat{\alpha}(e_2 - e_1) - k_1 e_1 \\ u_2(t) &= -\hat{\gamma} e_1 + y_1 y_3 - x_1 x_3 - k_2 e_2 \\ u_3(t) &= \hat{\beta} e_3 + y_1 y_2 - x_1 x_2 - k_3 e_3 \end{aligned} \tag{7}$$

where $\hat{\alpha}, \hat{\beta}$ and $\hat{\gamma}$ are estimates of α, β and γ, respectively, and $k_i, (i = 1, 2, 3)$ are positive constants.

Substituting (7) into (6), the error dynamics simplifies to

$$\begin{aligned} \dot{e}_1 &= (\alpha - \hat{\alpha})(e_2 - e_1) - k_1 e_1 \\ \dot{e}_2 &= (\gamma - \hat{\gamma}) e_1 - k_2 e_2 \\ \dot{e}_3 &= -(\beta - \hat{\beta}) e_3 - k_3 e_3 \end{aligned} \tag{8}$$

Let us now define the parameter estimation errors as

$$e_\alpha = \alpha - \hat{\alpha}, \quad e_\beta = \beta - \hat{\beta} \quad \text{and} \quad e_\gamma = \gamma - \hat{\gamma} \tag{9}$$

Substituting (9) into (8), we obtain the error dynamics as

$$\begin{aligned} \dot{e}_1 &= e_\alpha (e_2 - e_1) - k_1 e_1 \\ \dot{e}_2 &= e_\gamma e_1 - k_2 e_2 \\ \dot{e}_3 &= -e_\beta e_3 - k_3 e_3 \end{aligned} \tag{10}$$

For the derivation of the update law for adjusting the estimates of the parameters, the Lyapunov approach is used.

We consider the quadratic Lyapunov function defined by

$$V(e_1, e_2, e_3, e_\alpha, e_\beta, e_\gamma) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_\alpha^2 + e_\beta^2 + e_\gamma^2 \right), \tag{11}$$

which is a positive definite function on R^6.

We also note that

$$\dot{e}_\alpha = -\dot{\hat{\alpha}}, \; \dot{e}_\beta = -\dot{\hat{\beta}} \quad \text{and} \quad \dot{e}_\gamma = -\dot{\hat{\gamma}} \tag{12}$$

Differentiating (11) along the trajectories of (10) and using (12), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_\alpha \left[e_1(e_2 - e_1) - \dot{\hat{\alpha}} \right] + e_\beta \left[-e_3^2 - \dot{\hat{\beta}} \right] + e_\gamma \left[e_1 e_2 - \dot{\hat{\gamma}} \right] \tag{13}$$

In view of Eq. (13), the estimated parameters are updated by the following law:

$$\begin{aligned} \dot{\hat{\alpha}} &= e_1(e_2 - e_1) + k_4 e_\alpha \\ \dot{\hat{\beta}} &= -e_3^2 + k_5 e_\beta \\ \dot{\hat{\gamma}} &= e_1 e_2 + k_6 e_\gamma \end{aligned} \tag{14}$$

where k_4, k_5 and k_6 are positive constants.

Substituting (14) into (13), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_\alpha^2 - k_5 e_\beta^2 - k_6 e_\gamma^2 \tag{15}$$

which is a negative definite function on R^6.

Thus, by Lyapunov stability theory [35], it is immediate that the synchronization error $e_i, (i = 1, 2, 3)$ and the parameter estimation error $e_\alpha, e_\beta, e_\gamma$ decay to zero exponentially with time.

Hence, we have proved the following result.

Theorem 1. *The identical Pan systems (3) and (4) with unknown parameters are globally and exponentially synchronized via the adaptive control law (7), where the update law for the parameter estimates is given by (14) and* $k_i, (i = 1, 2, \ldots, 6)$ *are positive constants. Also, the parameter estimates* $\hat{\alpha}(t), \hat{\beta}(t)$ *and* $\hat{\gamma}(t)$ *exponentially converge to the original values of the parameters* α, β *and* γ*, respectively, as* $t \to \infty.$ ■

3.2 Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the hyperchaotic systems (3) and (4) with the adaptive control law (14) and the parameter update law (14) using MATLAB.

We take

$$k_i = 4 \text{ for } i = 1, 2, \ldots, 6.$$

For the Pan systems (3) and (4), the parameter values are taken as

$$\alpha = 10, \quad \beta = 8/3, \quad \gamma = 16$$

Suppose that the initial values of the parameter estimates are

$$\hat{\alpha}(0) = 3, \quad \hat{\beta}(0) = 5, \quad \hat{\gamma}(0) = 1$$

The initial values of the master system (3) are taken as

$$x_1(0) = 12, \quad x_2(0) = 5, \quad x_3(0) = 9$$

The initial values of the slave system (4) are taken as

$$y_1(0) = 7, \quad y_2(0) = 6, \quad y_3(0) = 10$$

Figure 3 depicts the global chaos synchronization of the identical Pan systems (3) and (4).

Figure 4 shows that the estimated values of the parameters, viz. $\hat{\alpha}(t), \hat{\beta}(t)$ and $\hat{\gamma}(t)$ converge exponentially to the system parameters $\alpha = 10,$ $\beta = 8/3$ and $\gamma = 16,$ as $t \to \infty.$

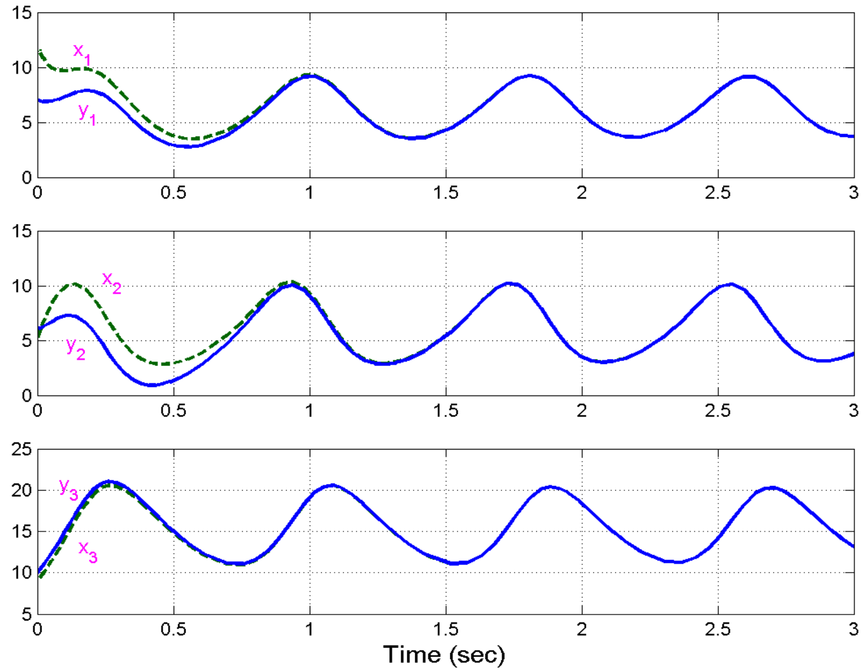

Figure 3. Complete Synchronization of Pan Systems

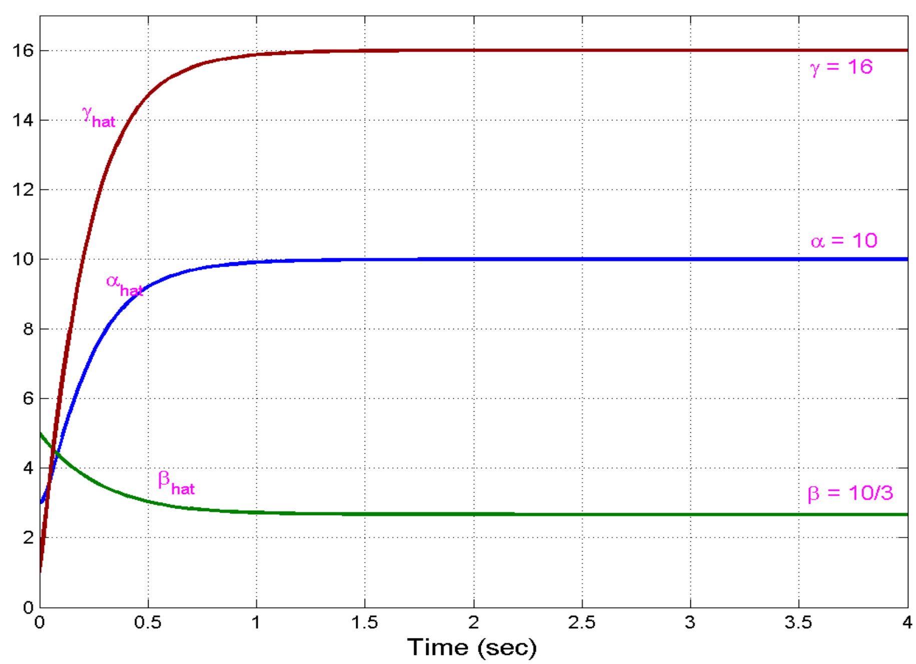

Figure 4. Parameter Estimates $\hat{\alpha}(t), \hat{\beta}(t), \hat{\gamma}(t)$

4. ADAPTIVE SYNCHRONIZATION OF IDENTICAL LÜ SYSTEMS

4.1 Theoretical Results

In this section, we deploy adaptive control to achieve new results for the global chaos synchronization of identical Lü systems ([34], 2002), where the parameters of the master and slave systems are unknown.

As the master system, we consider the Lü dynamics described by

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= cx_2 - x_1 x_3 \\ \dot{x}_3 &= -bx_3 + x_1 x_2 \end{aligned} \tag{16}$$

where x_1, x_2, x_3 are the state variables and a, b, c are unknown, real ,constant parameters of the system.

As the slave system, we consider the controlled Lü dynamics described by

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + u_1 \\ \dot{y}_2 &= cy_2 - y_1 y_3 + u_2 \\ \dot{y}_3 &= -by_3 + y_1 y_2 + u_3 \end{aligned} \tag{17}$$

where y_1, y_2, y_3 are the state variables and u_1, u_2, u_3 are the nonlinear controllers to be designed.

The chaos synchronization error is defined by

$$\begin{aligned} e_1 &= y_1 - x_1 \\ e_2 &= y_2 - x_2 \\ e_3 &= y_3 - x_3 \end{aligned} \tag{18}$$

The error dynamics is easily obtained as

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + u_1 \\ \dot{e}_2 &= ce_2 - y_1 y_3 + x_1 x_3 + u_2 \\ \dot{e}_3 &= -be_3 + y_1 y_2 - x_1 x_2 + u_3 \end{aligned} \tag{19}$$

Let us now define the adaptive control functions

$$\begin{aligned} u_1(t) &= -\hat{a}(e_2 - e_1) - k_1 e_1 \\ u_2(t) &= -\hat{c}e_2 + y_1 y_3 - x_1 x_3 - k_2 e_2 \\ u_3(t) &= \hat{b}e_3 - y_1 y_2 + x_1 x_2 - k_3 e_3 \end{aligned} \tag{20}$$

where $\hat{a}, \hat{b}$ and $\hat{c}$ are estimates of a, b and c, respectively, and $k_i, (i = 1, 2, 3)$ are positive constants.

Substituting (20) into (19), the error dynamics simplifies to

$$\begin{aligned}\dot{e}_1 &= (a-\hat{a})(e_2-e_1)-k_1e_1 \\ \dot{e}_2 &= (c-\hat{c})e_2-k_2e_2 \\ \dot{e}_3 &= -(b-\hat{b})e_3-k_3e_3\end{aligned} \tag{21}$$

Let us now define the parameter estimation errors as

$$e_a = a-\hat{a},\ e_b = b-\hat{b} \text{ and } e_c = c-\hat{c} \tag{22}$$

Substituting (22) into (21), we obtain the error dynamics as

$$\begin{aligned}\dot{e}_1 &= e_a(e_2-e_1)-k_1e_1 \\ \dot{e}_2 &= e_ce_2-k_2e_2 \\ \dot{e}_3 &= -e_be_3-k_3e_3\end{aligned} \tag{23}$$

We consider the quadratic Lyapunov function defined by

$$V(e_1,e_2,e_3,e_a,e_b,e_c) = \frac{1}{2}\left(e_1^2+e_2^2+e_3^2+e_a^2+e_b^2+e_c^2\right), \tag{24}$$

which is a positive definite function on R^6.

We also note that

$$\dot{e}_a = -\dot{\hat{a}},\ \dot{e}_b = -\dot{\hat{b}} \quad \text{and} \quad \dot{e}_c = -\dot{\hat{c}} \tag{25}$$

Differentiating (24) along the trajectories of (23) and using (25), we obtain

$$\dot{V} = -k_1e_1^2-k_2e_2^2-k_3e_3^2+e_a\left[e_1(e_2-e_1)-\dot{\hat{a}}\right]+e_b\left[-e_3^2-\dot{\hat{b}}\right]+e_c\left[e_2^2-\dot{\hat{c}}\right] \tag{26}$$

In view of Eq. (26), the estimated parameters are updated by the following law:

$$\begin{aligned}\dot{\hat{a}} &= e_1(e_2-e_1)+k_4e_a \\ \dot{\hat{b}} &= -e_3^2+k_5e_b \\ \dot{\hat{c}} &= e_2^2+k_6e_c\end{aligned} \tag{27}$$

where $k_i,\ (i=4,5,6)$ are positive constants.

Substituting (14) into (12), we obtain

$$\dot{V} = -k_1e_1^2-k_2e_2^2-k_3e_3^2-k_4e_a^2-k_5e_b^2-k_6e_c^2 \tag{28}$$

which is a negative definite function on R^6.

Thus, by Lyapunov stability theory [35], it is immediate that the synchronization error $e_i, (i = 1, 2, 3)$ and the parameter estimation error e_a, e_b, e_c decay to zero exponentially with time.

Hence, we have proved the following result.

Theorem 2. *The identical Lü systems (16) and (17) with unknown parameters are globally and exponentially synchronized via the adaptive control law (20), where the update law for the parameter estimates is given by (27) and* $k_i, (i = 1, 2, \ldots, 6)$ *are positive constants. Also, the parameter estimates* $\hat{a}(t), \hat{b}(t)$ *and* $\hat{c}(t)$ *exponentially converge to the original values of the parameters* a, b *and* c, *respectively, as* $t \to \infty$. ■

4.2 Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the chaotic systems (16) and (17) with the adaptive control law (20) and the parameter update law (27) using MATLAB.

We take $k_i = 4$ for $i = 1, 2, \ldots, 8.$

For the Lü systems (16) and (17), the parameter values are taken as

$$a = 36, \quad b = 3, \quad c = 20$$

Suppose that the initial values of the parameter estimates are

$$\hat{a}(0) = 16, \quad \hat{b}(0) = 8, \quad \hat{c}(0) = 4$$

The initial values of the master system (16) are taken as

$$x_1(0) = 11, \quad x_2(0) = 27, \quad x_3(0) = 5$$

The initial values of the slave system (17) are taken as

$$y_1(0) = 29, \quad y_2(0) = 15, \quad y_3(0) = 18$$

Figure 5 depicts the global chaos synchronization of the identical Lü systems (16) and (17).

Figure 6 shows that the estimated values of the parameters, *viz.* $\hat{a}(t), \hat{b}(t)$ and $\hat{c}(t)$ converge exponentially to the system parameters

$$a = 36, \; b = 3 \text{ and } \; c = 20$$

as $t \to \infty$.

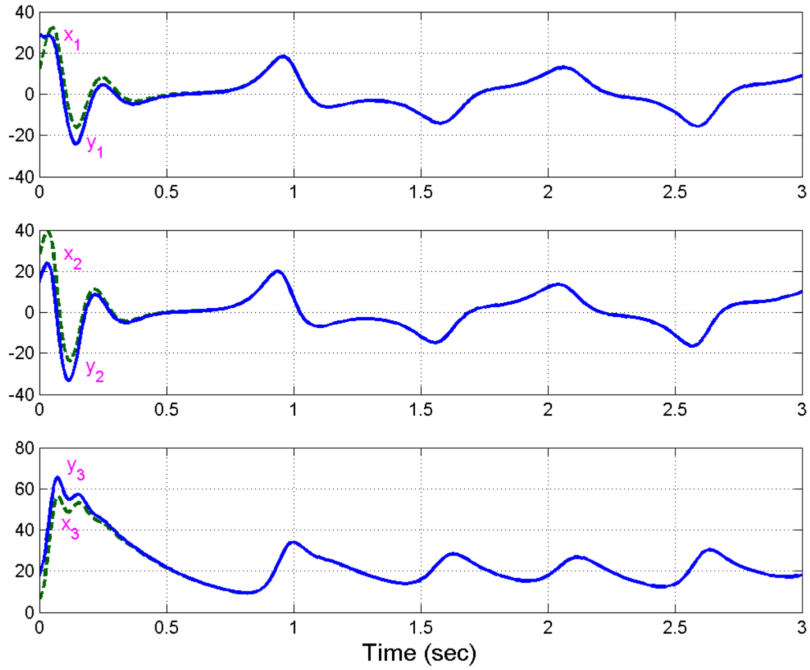

Figure 5. Complete Synchronization of Lü Systems

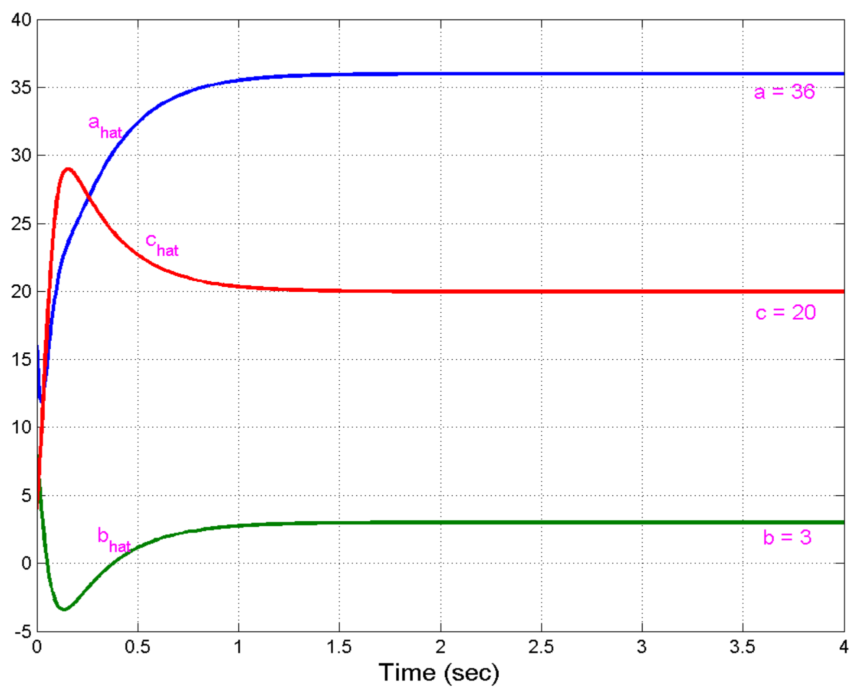

Figure 6. Parameter Estimates $\hat{a}(t)$, $\hat{b}(t)$, $\hat{c}(t)$

5. ADAPTIVE SYNCHRONIZATION OF PAN AND LÜ SYSTEMS

5.1 Theoretical Results

In this section, we discuss the global chaos synchronization of non-identical Pan system ([33], 2010) and Lü system ([34], 2002), where the parameters of the master and slave systems are unknown.

As the master system, we consider the Pan dynamics described by

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - x_1) \\ \dot{x}_2 &= \gamma x_1 - x_1 x_3 \\ \dot{x}_3 &= -\beta x_3 + x_1 x_2 \end{aligned} \tag{29}$$

where x_1, x_2, x_3 are the state variables and α, β, γ are unknown, real ,constant parameters of the system.

As the slave system, we consider the controlled Lü dynamics described by

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + u_1 \\ \dot{y}_2 &= c y_2 - y_1 y_3 + u_2 \\ \dot{y}_3 &= -b y_3 + y_1 y_2 + u_3 \end{aligned} \tag{30}$$

where y_1, y_2, y_3 are the state variables , a, b, c are unknown, real, constant parameters of the system and u_1, u_2, u_3 are the nonlinear controllers to be designed.

The synchronization error is defined by

$$e_i = y_i - x_i, \quad (i = 1, 2, 3) \tag{31}$$

The error dynamics is easily obtained as

$$\begin{aligned} \dot{e}_1 &= a(y_2 - y_1) - \alpha(x_2 - x_1) + u_1 \\ \dot{e}_2 &= c y_2 - \gamma x_1 - y_1 y_3 + x_1 x_3 + u_2 \\ \dot{e}_3 &= -b y_3 + \beta x_3 + y_1 y_2 - x_1 x_2 + u_3 \end{aligned} \tag{32}$$

Let us now define the adaptive control functions

$$\begin{aligned} u_1(t) &= -\hat{a}(y_2 - y_1) + \hat{\alpha}(x_2 - x_1) - k_1 e_1 \\ u_2(t) &= -\hat{c} y_2 + \hat{\gamma} x_1 + y_1 y_3 - x_1 x_3 - k_2 e_2 \\ u_3(t) &= \hat{b} y_3 - \hat{\beta} x_3 - y_1 y_2 + x_1 x_2 - k_3 e_3 \end{aligned} \tag{33}$$

where $\hat{a}, \hat{b}, \hat{c}, \hat{\alpha}, \hat{\beta}$ and $\hat{\gamma}$ are estimates of a, b, c, α, β and γ, respectively, and k_1, k_2, k_3 are positive constants.

Substituting (33) into (32), the error dynamics simplifies to

$$\begin{aligned}\dot{e}_1 &= (a-\hat{a})(y_2-y_1)-(\alpha-\hat{\alpha})(x_2-x_1)-k_1e_1\\ \dot{e}_2 &= (c-\hat{c})y_2-(\gamma-\hat{\gamma})x_1-k_2e_2\\ \dot{e}_3 &= -(b-\hat{b})y_3+(\beta-\hat{\beta})x_3-k_3e_3\end{aligned} \tag{34}$$

Let us now define the parameter estimation errors as

$$e_a = a-\hat{a},\;\; e_b = b-\hat{b},\; e_c = c-\hat{c},\; e_\alpha = \alpha-\hat{\alpha},\; e_\beta = \beta-\hat{\beta},\; e_\gamma = \gamma-\hat{\gamma} \tag{35}$$

Substituting (35) into (34), we obtain the error dynamics as

$$\begin{aligned}\dot{e}_1 &= e_a(y_2-y_1)-e_\alpha(x_2-x_1)-k_1e_1\\ \dot{e}_2 &= e_c y_2-e_\gamma x_1-k_2e_2\\ \dot{e}_3 &= -e_b y_3+e_\beta x_3-k_3e_3\end{aligned} \tag{36}$$

We consider the quadratic Lyapunov function defined by

$$V = \frac{1}{2}\left(e_1^2+e_2^2+e_3^2+e_a^2+e_b^2+e_c^2+e_\alpha^2+e_\beta^2+e_\gamma^2\right), \tag{37}$$

which is a positive definite function on R^9.

We also note that

$$\dot{e}_a = -\dot{\hat{a}},\; \dot{e}_b = -\dot{\hat{b}},\; \dot{e}_c = -\dot{\hat{c}},\; \dot{e}_\alpha = -\dot{\hat{\alpha}},\; \dot{e}_\beta = -\dot{\hat{\beta}},\; \dot{e}_\gamma = -\dot{\hat{\gamma}} \tag{38}$$

Differentiating (37) along the trajectories of (36) and using (38), we obtain

$$\begin{aligned}\dot{V} = &-k_1e_1^2-k_2e_2^2-k_3e_3^2+e_a\left[e_1(y_2-y_1)-\dot{\hat{a}}\right]+e_b\left[-e_3y_3-\dot{\hat{b}}\right]+e_c\left[e_2y_2-\dot{\hat{c}}\right]\\ &+e_\alpha\left[-e_1(x_2-x_1)-\dot{\hat{\alpha}}\right]+e_\beta\left[e_3x_3-\dot{\hat{\beta}}\right]+e_\gamma\left[-e_2x_1-\dot{\hat{\gamma}}\right]\end{aligned} \tag{39}$$

In view of Eq. (39), the estimated parameters are updated by the following law:

$$\begin{aligned}\dot{\hat{a}} &= e_1(y_2-y_1)+k_4e_a, & \dot{\hat{\alpha}} &= -e_1(x_2-x_1)+k_7e_\alpha\\ \dot{\hat{b}} &= -e_3y_3+k_5e_b, & \dot{\hat{\beta}} &= e_3x_3+k_8e_\beta\\ \dot{\hat{c}} &= e_2y_2+k_6e_c, & \dot{\hat{\gamma}} &= -e_2x_1+k_9e_\gamma\end{aligned} \tag{40}$$

where $k_i, (i = 4,\ldots,9)$ are positive constants.

Substituting (40) into (39), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_a^2 - k_5 e_b^2 - k_6 e_c^2 - k_7 e_\alpha^2 - k_8 e_\beta^2 - k_9 e_\gamma^2 \tag{41}$$

which is a negative definite function on R^9.

Thus, by Lyapunov stability theory [35], it is immediate that the synchronization error $e_i, (i = 1,2,3)$ and all the parameter estimation errors decay to zero exponentially with time.

Hence, we have proved the following result.

Theorem 3. *The non-identical Pan system (29) and Lü system (30) with unknown parameters are globally and exponentially synchronized via the adaptive control law (33), where the update law for the parameter estimates is given by (40) and* $k_i, (i = 1,2,\ldots,9)$ *are positive constants. Also, the parameter estimates* $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{\alpha}(t), \hat{\beta}(t)$ *and* $\hat{\gamma}(t)$ *exponentially converge to the original values of the parameters* a, b, c, α, β *and* γ, *respectively, as* $t \to \infty$. ■

5.2 Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the hyperchaotic systems (29) and (30) with the adaptive control law (33) and the parameter update law (40) using MATLAB.

We take $k_i = 4$ for $i = 1,2,\ldots,9.$

For the Lü and Pan systems, the parameters of the systems are chosen so that the systems are chaotic (see Section 2).

Suppose that the initial values of the parameter estimates are

$$\hat{a}(0) = 2,\ \hat{b}(0) = 4,\ \hat{c}(0) = 10,\ \hat{\alpha}(0) = 5,\ \hat{\beta}(0) = 9,\ \hat{\gamma}(0) = 7$$

The initial values of the master system (29) are taken as

$$x_1(0) = 17,\ \ x_2(0) = 24,\ \ x_3(0) = 12$$

The initial values of the slave system (30) are taken as

$$y_1(0) = 26,\ \ y_2(0) = 5,\ \ y_3(0) = 11$$

Figure 7 depicts the global chaos synchronization of Pan and Lü systems.

Figure 8 shows that the estimated values of the parameters, viz. $\hat{a}(t),\ \hat{b}(t),\ \hat{c}(t),\ \hat{\alpha}(t),\ \hat{\beta}(t)$ and $\hat{\gamma}(t)$ converge exponentially to the system parameters $a = 36,\ b = 3,\ c = 20,\ \alpha = 10,\ \beta = 8/3$ and $\gamma = 16,$ respectively, as $t \to \infty.$

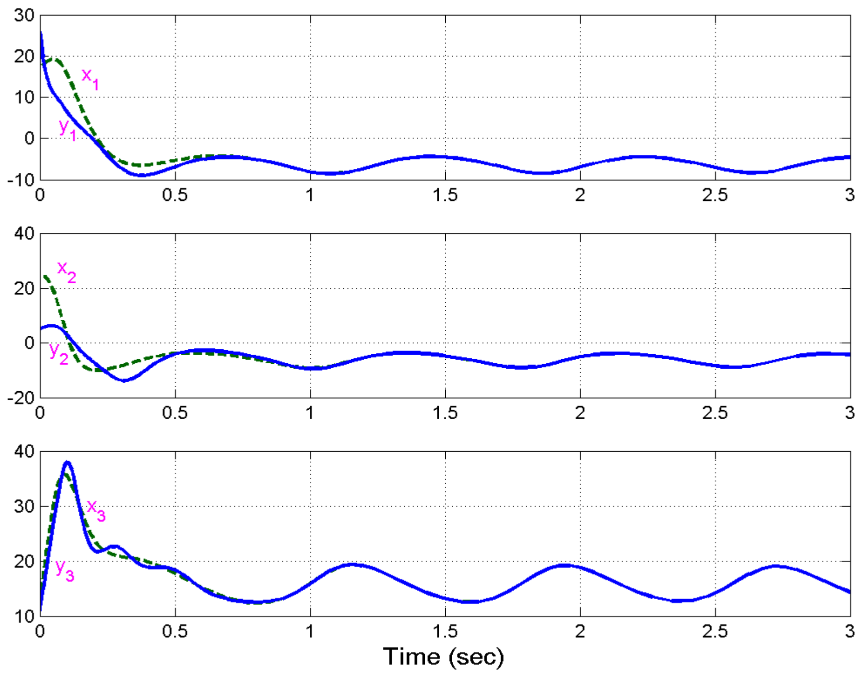

Figure 7. Complete Synchronization of Pan and Lü Systems

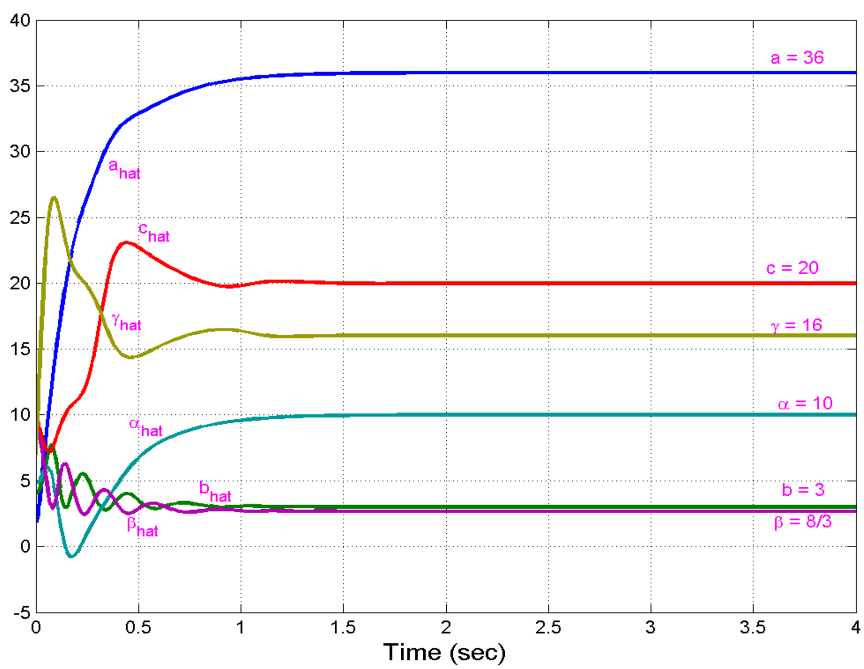

Figure 8. Parameter Estimates $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{\alpha}(t), \hat{\beta}(t), \hat{\gamma}(t)$

6. CONCLUSIONS

In this paper, we have derived new results for the adaptive synchronization of identical Pan systems (2010), identical Lü systems (2002) and non-identical Pan and Lü systems with unknown parameters. The adaptive synchronization results derived in this paper are established using adaptive control theory and Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the adaptive control method is a very suitable for achieving global chaos synchronization for the uncertain chaotic systems addressed in this paper. Numerical simulations are shown to validate and demonstrate the effectiveness of the adaptive synchronization schemes derived in this paper for the global chaos synchronization of the chaotic systems addressed in this paper.

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