ADAPTIVE CONTROL AND SYNCHRONIZATION OF SPROTT-I SYSTEM WITH UNKNOWN PARAMETERS

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ABSTRACT

This paper derives new results for the adaptive control and synchronization design of the Sprott-I chaotic system (1994), when the system parameters are unknown. First, we build an adaptive controller to stabilize the Sprott-I chaotic system to its unstable equilibrium at the origin. Then we build an adaptive synchronizer to achieve global chaos synchronization of the identical Sprott-I chaotic systems with unknown parameters. The results derived for adaptive stabilization and adaptive synchronization for the Sprott-I chaotic system have been established using adaptive control theory and Lyapunov stability theory. Numerical simulations have been shown to demonstrate the effectiveness of the adaptive control and synchronization schemes derived in this paper for the Sprott-I chaotic system.

KEYWORDS

Adaptive Control, Chaos, Chaotic Systems, Synchronization, Sprott-I System.

1. INTRODUCTION

Chaotic systems are nonlinear dynamical systems which are extremely sensitive to changes in initial conditions and also exhibit random-like behaviour in its deterministic motion. Experimentally, chaos was first discovered by Lorenz ([1], 1963) while he was simulating weather models. A chaotic system simpler than the Lorenz system was proposed by Rössler ([2], 1976). The theoretical equations of the Rössler system were later found to be useful in modelling equilibrium in chemical reactions.

The control of chaotic systems is to design state feedback control laws that stabilizes the chaotic systems around the unstable equilibrium points. Active control technique is used when the system parameters are known and adaptive control technique is used when the system parameters are unknown [3-4].

Synchronization of chaotic systems is a phenomenon that may occur when two or more chaotic attractors are coupled or when a hyperchaotic attractor drives another hyperchaotic attractor. In the last two decades, there has been significant interest in the literature on the synchronization of chaotic and hyperchaotic systems [5-16].

In 1990, Pecora and Carroll [5] introduced a method to synchronize two identical chaotic systems and showed that it was possible for some chaotic systems to be completely synchronized. From then on, chaos synchronization has been widely explored in a variety of fields including physical systems [6], chemical systems [7], ecological systems [8], secure communications [9-10], etc.

The pioneering work by Pecora and Carroll (1990) has been followed by a variety of impressive approaches in the literature such as the sampled-data feedback method [11], OGY method [12], time-delay feedback method [13], backstepping method [14], active control method [15-20], adaptive control method [21-25], sliding mode control method [26-28], etc.

This paper is organized as follows. In Section 2, we give a description of the Sprott-I chaotic system (Sprott, [29], 1994). In Section 3, we derive results for the adaptive control of Sprott-I chaotic system with unknown parameters. In Section 4, we derive results for the adaptive synchronization of the identical Sprott-I chaotic systems with unknown parameters. Section 5 contains a summary of the main results derived in this paper.

2. SYSTEM DESCRIPTION

The Sprott-I system ([29], 1994) is described by the 3D dynamics

$$\begin{aligned} \dot{x}_1 &= -a x_2 \\ \dot{x}_2 &= x_1 + b x_3 \\ \dot{x}_3 &= c x_1 + x_2^2 - x_3 \end{aligned} \tag{1}$$

where x_1, x_2, x_3 are the state variables of the system and a, b, c are constant, positive parameters of the system.

The system (1) is chaotic when the parameter values are taken as

$$a = 0.2, \quad b = 1 \text{ and } c = 1 \tag{2}$$

Figure 1 describes the strange attractor of the Sprott-I chaotic system (1).

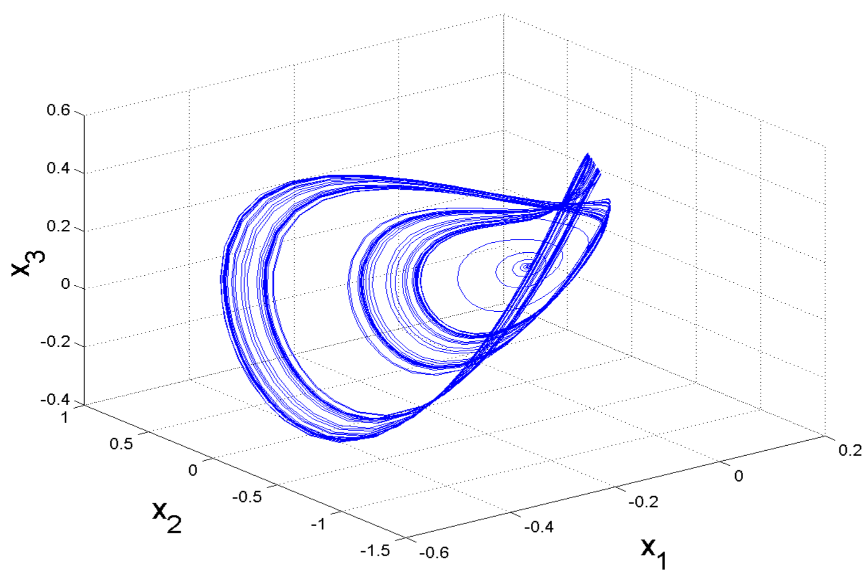

Figure 1. The Strange Attractor of the Sprott-I Chaotic System

When the parameter values are taken as in (2) for the Sprott-I chaotic system (1), the system linearization matrix at the equilibrium point $E_0 = (0,0,0)$ is given by

$$A = \begin{bmatrix} 0 & -0.2 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

which has the eigenvalues

$$\lambda_1 = -1.1345, \ \lambda_2 = 0.0672 + 0.5900\, i \quad \text{and} \quad \lambda_3 = 0.0672 - 0.5900\, i$$

Since λ_2 and λ_3 are unstable eigenvalues of A, it follows from Lyapunov stability theory [30] that the Sprott-I system (1) is unstable at the equilibrium point $E_0 = (0,0,0)$.

3. ADAPTIVE CONTROL OF THE SPROTT-I CHAOTIC SYSTEM

3.1 Theoretical Results

In this section, we design adaptive control law for globally stabilizing the Sprott-I system (1994), when the parameter values are unknown.

Thus, we consider the controlled Sprott-I system, which is described by the 3D dynamics

$$\begin{aligned} \dot{x}_1 &= -ax_2 + u_1 \\ \dot{x}_2 &= x_1 + bx_3 + u_2 \\ \dot{x}_3 &= cx_1 + x_2^2 - x_3 + u_3 \end{aligned} \tag{3}$$

where u_1, u_2 and u_3 are feedback controllers to be designed using the states x_1, x_2, x_3 and estimates $\hat{a}, \hat{b}, \hat{c}$ of the unknown parameters a, b, c of the system.

In order to ensure that the controlled system (3) globally converges to the origin asymptotically, we consider the following adaptive control functions

$$\begin{aligned} u_1 &= \hat{a}x_2 - k_1 x_1 \\ u_2 &= -x_1 - \hat{b}x_3 - k_2 x_2 \\ u_3 &= -\hat{c}x_1 - x_2^2 + x_3 - k_3 x_3 \end{aligned} \tag{4}$$

where $\hat{a}, \hat{b}$ and $\hat{c}$ are estimates of the parameters a, b and c, respectively, and $k_i, (i = 1,2,3)$ are positive constants.

Substituting the control law (4) into the controlled Sprott-I dynamics (3), we obtain

$$\begin{aligned}\dot{x}_1 &= -(a-\hat{a})\,x_2 - k_1\,x_1 \\ \dot{x}_2 &= (b-\hat{b})x_3 - k_2\,x_2 \\ \dot{x}_3 &= -(c-\hat{c})\,x_1 - k_3\,x_3\end{aligned} \tag{5}$$

Let us now define the parameter errors as

$$e_a = a - \hat{a}, \quad e_b = b - \hat{b} \text{ and } e_c = c - \hat{c} \tag{6}$$

Using (6), the closed-loop dynamics (5) can be written compactly as

$$\begin{aligned}\dot{x}_1 &= -e_a\,x_2 - k_1\,x_1 \\ \dot{x}_2 &= e_b x_3 - k_2\,x_2 \\ \dot{x}_3 &= -e_c\,x_1 - k_3\,x_3\end{aligned} \tag{7}$$

For the derivation of the update law for adjusting the parameter estimates $\hat{a}, \hat{b}$ and $\hat{c}$, the Lyapunov approach is used.

Consider the quadratic Lyapunov function

$$V(x_1, x_2, x_3, e_a, e_b, e_c) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + e_a^2 + e_b^2 + e_c^2\right) \tag{8}$$

which is a positive definite function on R^6.

Note also that

$$\dot{e}_a = -\dot{\hat{a}}, \quad \dot{e}_b = -\dot{\hat{b}}, \quad \dot{e}_c = -\dot{\hat{c}} \tag{9}$$

Differentiating V along the trajectories of (7) and using (9), we obtain

$$\dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 + e_a\left(-x_1 x_2 - \dot{\hat{a}}\right) + e_b\left(x_2 x_3 - \dot{\hat{b}}\right) + e_c\left(x_1 x_3 - \dot{\hat{c}}\right) \tag{10}$$

In view of Eq. (10), the estimated parameters are updated by the following law:

$$\begin{aligned}\dot{\hat{a}} &= -x_1 x_2 + k_4 e_a \\ \dot{\hat{b}} &= x_2 x_3 + k_5 e_b \\ \dot{\hat{c}} &= x_1 x_3 + k_6 e_c\end{aligned} \tag{11}$$

where k_4, k_5 and k_6 are positive constants.

Next, we prove the following result.

Theorem 1. *The controlled Sprott-I system (1) with unknown parameters is globally and exponentially stabilized for all initial conditions* $x(0) \in R^3$ *by the adaptive control law (4), where the update law for the parameters is given by (11) and* $k_i,\ (i = 1, \ldots, 6)$ *are positive constants.*

Proof. Substituting (11) into (10), we get

$$\dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 - k_4 e_a^2 - k_5 e_b^2 - k_6 e_c^2 \tag{12}$$

which is a negative definite function on R^6.

Thus, by Lyapunov stability theory [30], it is immediate that the controlled Sprott-I system (7) is globally exponentially stable and also that the parameter estimation errors e_a, e_b, e_c exponentially converge to zero with time.

This completes the proof. ■

3.2 Numerical Results

For the numerical simulations, the fourth order Runge-Kutta method is used to solve the chaotic system (3) with the adaptive control law (4) and the parameter update law (11).

The parameters of the Sprott-I system (3) are selected as

$$a = 0.2,\quad b = 1 \text{ and } c = 1.$$

For the adaptive and update laws, we take

$$k_i = 5,\quad (i = 1, 2, \ldots, 6).$$

Suppose that the initial values of the estimated parameters are

$$\hat{a}(0) = 4,\quad \hat{b}(0) = 12,\quad \hat{c}(0) = 9$$

The initial state of the controlled Sprott-I system (3) is taken as

$$x_1(0) = 9,\quad x_2(0) = -14,\quad x_3(0) = 10$$

When the adaptive control law (4) and the parameter update law (11) are used, the controlled modified Sprott-I system converges to the equilibrium $E_0 = (0, 0, 0)$ exponentially as shown in Figure 2.

The time-history of the parameter estimates is shown in Figure 3.

The time-history of the parameter estimation errors is shown in Figure 4.

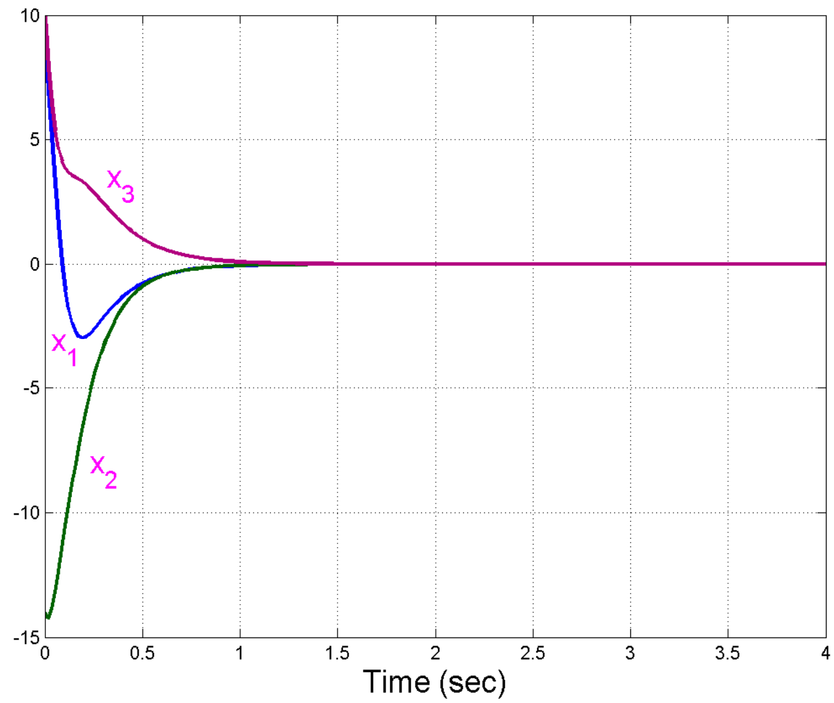

Figure 2. Time Responses of the Controlled Sprott-I System

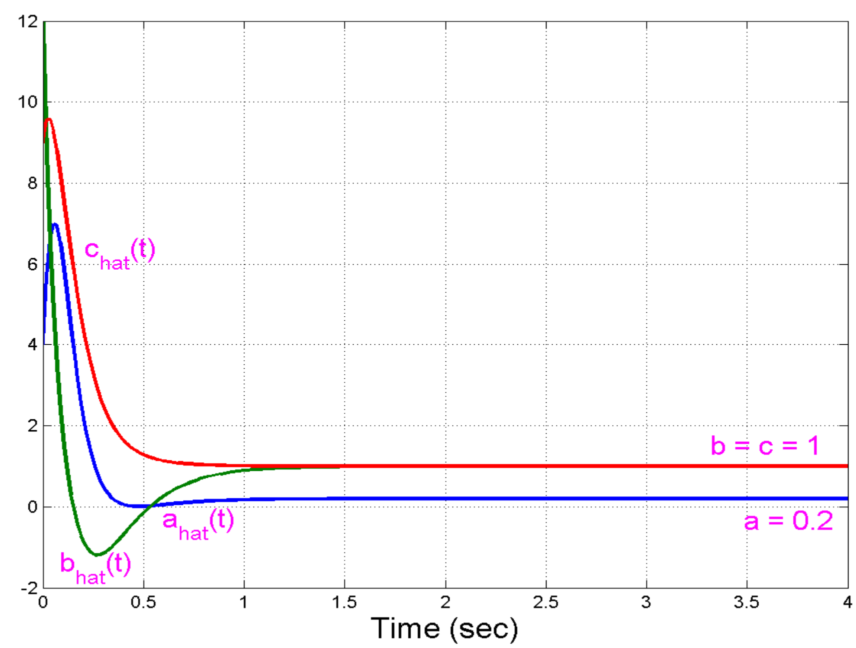

Figure 3. Time-History of the Parameter Estimates $\hat{a}(t)$, $\hat{b}(t)$, $\hat{c}(t)$

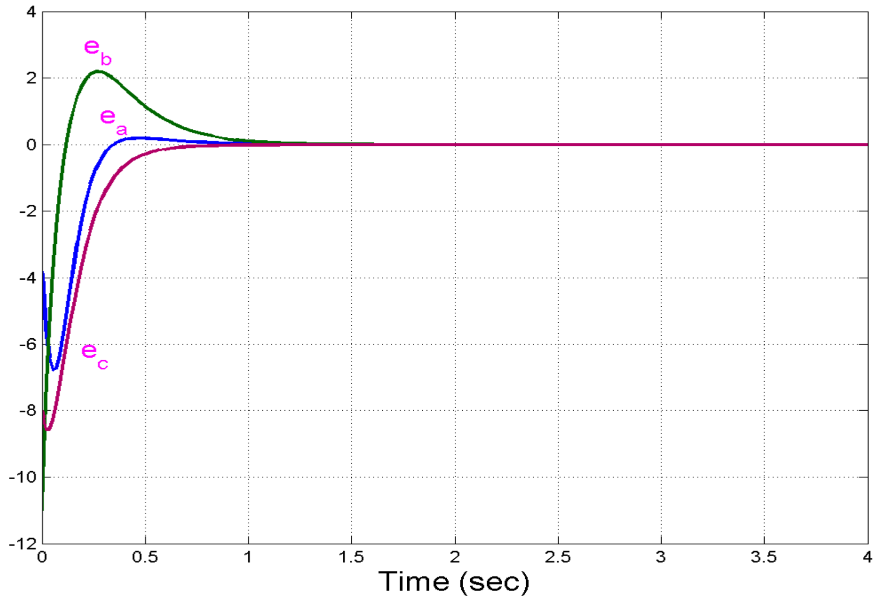

Figure 4. Time-History of the Parameter Estimates e_a, e_b, e_c

4. Adaptive Synchronization of Identical Sprott-I Chaotic Systems

4.1 Theoretical Results

In this section, we discuss the adaptive synchronization of identical Sprott-I chaotic systems (1994) with unknown parameters.

As the master system, we consider the Sprott-I dynamics described by

$$\begin{aligned} \dot{x}_1 &= -ax_2 \\ \dot{x}_2 &= x_1 + bx_3 \\ \dot{x}_3 &= cx_1 + x_2^2 - x_3 \end{aligned} \tag{13}$$

where $x_i, (i = 1, 2, 3)$ are the state variables and a, b, c are unknown system parameters.

As the slave system, we consider the controlled Sprott-I system described by

$$\begin{aligned} \dot{y}_1 &= -ay_2 + u_1 \\ \dot{y}_2 &= y_1 + by_3 + u_2 \\ \dot{y}_3 &= cy_1 + y_2^2 - y_3 + u_3 \end{aligned} \tag{14}$$

where $y_i, (i = 1, 2, 3)$ are the state variables and $u_i, (i = 1, 2, 3)$ are adaptive controllers to be designed.

The synchronization error is defined by

$$\begin{aligned} e_1 &= y_1 - x_1 \\ e_2 &= y_2 - x_2 \\ e_3 &= y_3 - x_3 \end{aligned} \tag{15}$$

Then the error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= -ae_2 + u_1 \\ \dot{e}_2 &= e_1 + be_3 + u_2 \\ \dot{e}_3 &= ce_1 - e_3 + y_2^2 - x_2^2 + u_3 \end{aligned} \tag{16}$$

Let us now define the adaptive control functions $u_1(t), u_2(t), u_3(t)$ as

$$\begin{aligned} u_1 &= -\hat{a}e_2 - k_1 e_1 \\ u_2 &= -e_1 - \hat{b}e_3 - k_2 e_2 \\ u_3 &= -\hat{c}e_1 + e_3 - y_2^2 + x_2^2 - k_3 e_3 \end{aligned} \tag{17}$$

where $\hat{a}, \hat{b}$ and $\hat{c}$ are estimates of the parameters a, b and $c,$ respectively, and $k_i, (i = 1, 2, 3)$ are positive constants.

Substituting the control law (17) into (16), we obtain the error dynamics as

$$\begin{aligned} \dot{e}_1 &= -(a - \hat{a})e_2 - k_1 e_1 \\ \dot{e}_2 &= (b - \hat{b})e_3 - k_2 e_2 \\ \dot{e}_3 &= (c - \hat{c})e_1 - k_3 e_3 \end{aligned} \tag{18}$$

Let us now define the parameter errors as

$$e_a = a - \hat{a}, \quad e_b = b - \hat{b}, \quad e_c = c - \hat{c} \tag{19}$$

Substituting (19) into (18), the error dynamics simplifies to

$$\begin{aligned} \dot{e}_1 &= -e_a e_2 - k_1 e_1 \\ \dot{e}_2 &= e_b e_3 - k_2 e_2 \\ \dot{e}_3 &= e_c e_1 - k_3 e_3 \end{aligned} \tag{20}$$

Consider the quadratic Lyapunov function

$$V(e_1, e_2, e_3, e_a, e_b, e_c) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2 + e_c^2\right) \tag{21}$$

which is a positive definite function on R^6.

Note also that

$$\dot{e}_a = -\dot{\hat{a}}, \quad \dot{e}_b = -\dot{\hat{b}}, \quad \dot{e}_c = -\dot{\hat{c}} \tag{22}$$

Differentiating V along the trajectories of (20) and using (22), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_a\left(-e_1 e_2 - \dot{\hat{a}}\right) + e_b\left(e_2 e_3 - \dot{\hat{b}}\right) + e_c\left(e_1 e_3 - \dot{\hat{c}}\right) \tag{23}$$

In view of Eq. (23), the estimated parameters are updated by the following law:

$$\begin{aligned} \dot{\hat{a}} &= -e_1 e_2 + k_4 e_a \\ \dot{\hat{b}} &= e_2 e_3 + k_5 e_b \\ \dot{\hat{c}} &= e_1 e_3 + k_6 e_c \end{aligned} \tag{24}$$

where k_4, k_5 and k_6 are positive constants.

Theorem 2. *The identical Sprott-I systems (13) and (14) with unknown parameters are globally and exponentially synchronized for all initial conditions by the adaptive control law (17), where the update law for parameters is given by (24) and* $k_i, (i = 1, \ldots, 6)$ *are positive constants.*

Proof. Substituting (24) into (23), we get

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_a^2 - k_5 e_b^2 - k_6 e_c^2 \tag{25}$$

From (25), we find that $\dot{V}$ is a negative definite function on R^6.

Thus, by Lyapunov stability theory [30], it is immediate that the synchronization error and the parameter error decay to zero exponentially with time for all initial conditions. ■

4.2 Numerical Results

For the numerical simulations, the fourth order Runge-Kutta method is used to solve the two systems of differential equations (13) and (14) with the adaptive control law (17) and the parameter update law (24).

We take the parameter values as in the chaotic case, viz.

$$a = 0.2, \quad b = 1, \quad c = 1$$

We take the positive constants $k_i, \ (i = 1, \ldots, 8)$ as

$$k_i = 5 \ \text{ for } \ i = 1, 2, \ldots, 6.$$

Suppose that the initial values of the estimated parameters are

$$\hat{a}(0) = 6, \quad \hat{b}(0) = 12, \quad \hat{c}(0) = 8$$

We take the initial values of the master system (13) as

$$x_1(0) = 12, \quad x_2(0) = -15, \quad x_3(0) = 4$$

We take the initial values of the slave system (14) as

$$y_1(0) = 23, \quad y_2(0) = 10, \quad y_3(0) = -7$$

Figure 5 shows the adaptive chaos synchronization of the identical Sprott-I systems.

Figure 6 shows the time-history of the parameter estimates $\hat{a}(t), \hat{b}(t), \hat{c}(t)$. From this figure, it is clear that the parameter estimates converge to the chosen values of a, b, c respectively.

Figure 7 shows the time-history of the parameter estimation errors e_a, e_b, e_c.

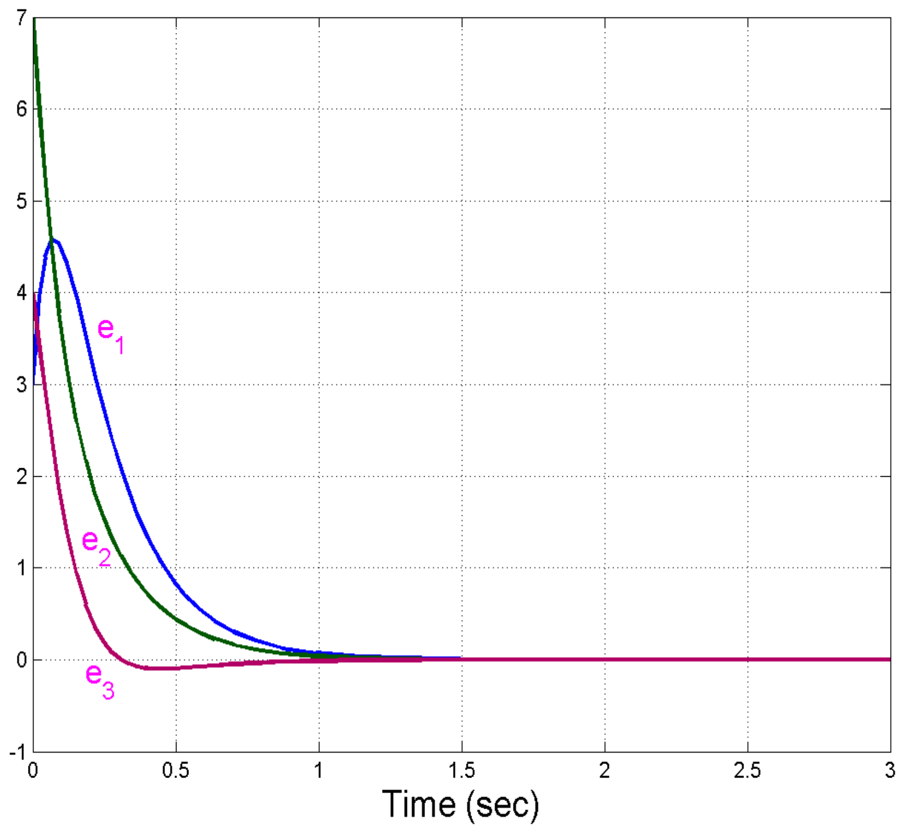

Figure 5. Adaptive Synchronization of the Sprott-I Systems

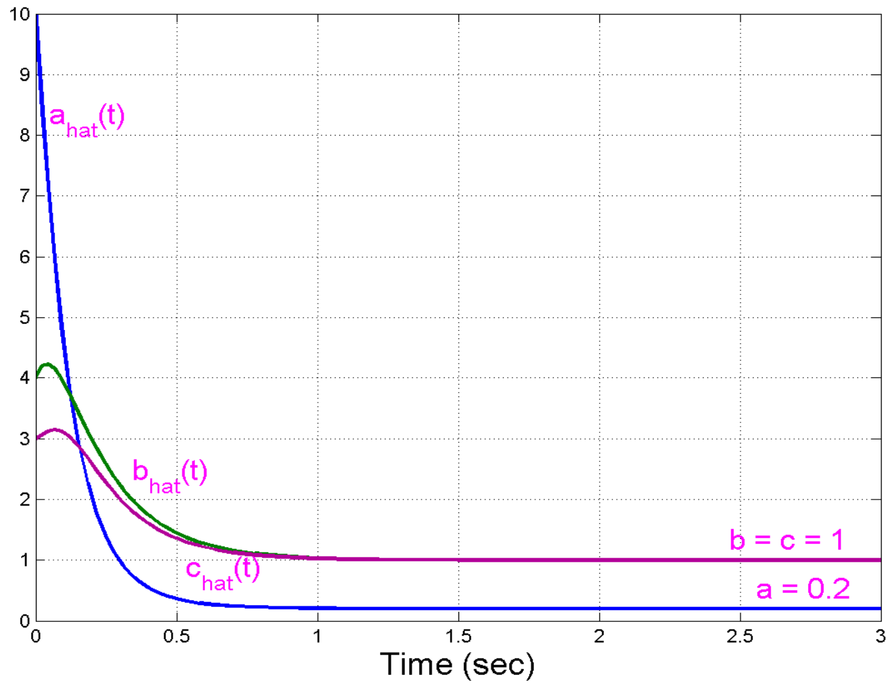

Figure 6. Time-History of the Parameter Estimates $\hat{a}(t), \hat{b}(t), \hat{c}(t)$

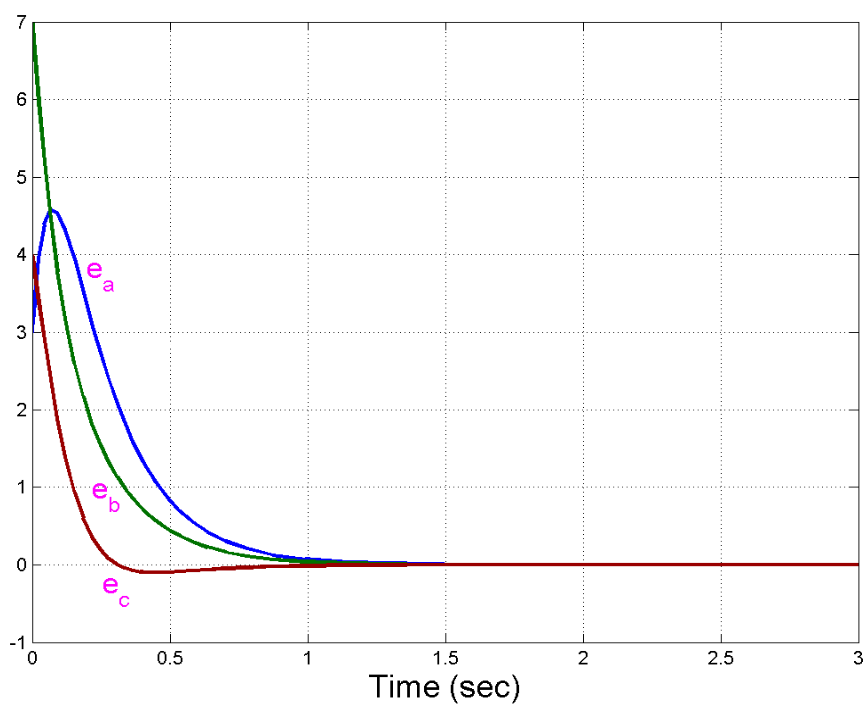

Figure 7. Time-History of the Parameter Estimation Error e_a, e_b, e_c

5. CONCLUSIONS

In this paper, we applied adaptive control theory for the stabilization and synchronization of the Sprott-I system (1994) with unknown system parameters. First, we designed adaptive control laws to stabilize the Sprott-I system to its unstable equilibrium point at the origin based on the adaptive control theory and Lyapunov stability theory. Then we derived adaptive synchronization scheme and update law for the estimation of system parameters for the identical Sprott-I systems with unknown parameters. Our synchronization schemes were established using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the proposed adaptive control method is very effective and convenient to achieve chaos control and synchronization of the Sprott-I chaotic system. Numerical simulations are shown to demonstrate the effectiveness of the proposed adaptive stabilization and synchronization schemes.

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