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ERGODIC CAPACITY ANALYSIS OF COOPERATIVE AMPLIFY-AND-FORWARD RELAY NETWORKS OVER RICE AND NAKAGAMI FADING CHANNELS

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ABSTRACT

This article investigates the efficacy of a novel moment generating function (MGF) based analytical framework for calculating the ergodic channel capacities of cooperative dual-hop amplify-and-forward (CAF) relay networks under three distinct source-adaptive transmission policies in Rice and Nakagami-m fading environments. The proposed analytical approach relies on a new exponential-type integral representation for the logarithmic function $\ln(\gamma)=\int_0^\infty x^{-1}(e^{-2x}-e^{-2x\gamma})dx,\ \gamma>0,$ *that facilitates the task of averaging it over the probability density function (PDF) of end-to-end signal-to-noise ratio (SNR) in a myriad of stochastic fading environments. We also resort to a well-known bounding technique to derive closed-form formulas for the upper and lower bounds for the MGF of dual-hop CAF relayed path SNR in Rice and Nakagami-m channels (since exact evaluation of the MGF of half-harmonic mean SNR in Rice fading is known to be very complicated and numerically unstable). These two attributes dramatically reduces the computational complexity of evaluating the ergodic capacities of CAF multi-relay networks with the optimal joint power-and-rate adaptation (when the channel side information is available at both the transmitter and the receiver) and the optimal rate-adaptation with a constant transmit power (when channel side information is available at the receiver only) policies.*

KEYWORDS

cooperative diversity, source-adaptive transmission strategies, ergodic capacity, moment generating function technique, amplify-and-forward relay networks

1. INTRODUCTION

The exploding demand for ubiquitous computing and communications in the last decade has exemplified the need for significant improvements in the spectrum utilization efficiency, energy efficiency and the data rates supported by the emerging wireless systems, while ensuring the integrity of data transmission over the noisy and time-varying wireless links. Link adaptation (wherein the source signalling rate, transmitted power level, coding rate, constellation size, packet length, and/or other system parameters are "matched' to the prevailing channel condition) is known to be an effective communication technique for increasing the data rate and spectral efficiency of wireless data centric networks. In recent years, a new communication paradigm known as "cooperative communications" has received a considerable attention owing to its ability to exploit the broadcast nature of wireless transmissions for harnessing a new form of spatial diversity to combat the deleterious effects of multipath fading [1]. Specifically, this form of "user cooperation diversity" facilitates an evolutionary path for reaping the benefits of multiple-input-multiple-output (MIMO) antenna technology with existing small form-factor hand-held devices that are not equipped with an antenna array (e.g., cell-phones, sensor nodes).

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The cooperative relaying architecture also offers a modular and flexible solution to meet a prescribed design objective (e.g., data rate, error rate, energy constraint, etc.) and quality of service assurance by enabling the source node to tap into the available resources of local neighbouring nodes to increase its throughput, range, reliability, and covertness. This feature makes it very attractive for a wide range of applications including battlefield communications, first-responder and disaster management networks, cellular communications, wireless sensor networks, vehicular/mobile ad-hoc networks, among many others.

An intermediate node (i.e., relay) in a cooperative relay network may either amplify what it receives (in case of amplify-and-forward relaying protocol) or digitally decodes, and re-encodes the source information (in case of decode-and-forward relaying protocol) before re-transmitting it to the destination node. Other variations of cooperative relaying strategies include compress-and-forward, opportunistic, incremental, variable-gain and fixed-gain (either blind or semi-blind) relaying that are implemented based on the availability of channel side-information (CSI) and the number of active participating nodes for information relaying. In this article, we primarily focus on variable-gain cooperative amplify-and-forward (CAF) relaying protocol because it does not require "sophisticated" transceivers at the relays, although our ergodic capacity analysis may be extended to other categories and variations of cooperative diversity schemes if the moment generating function (MGF) of end-to-end signal-to-noise ratio (SNR) is available. While this protocol can achieve a full diversity by forming a virtual antenna array, there is a loss of spectral efficiency due to its inherent half-duplex operation. This penalty, however, could be "recovered" to some extent by combining the cooperative diversity with a link adaptation mechanism and/or resorting to an opportunistic relaying strategy. The efficacy of integrating link adaptation schemes into the cooperative relay network would be of interest to the system developers of the emerging IEEE 802.16 wireless networks (i.e., the current IEEE 802.16e systems employ adaptive modulation, while the emerging IEEE 802.16j standard specifies the use of cooperative diversity in its multi-hop relay architecture).

But the art of adaptive link layer in cooperative wireless networks is still in its infancy especially when optimized in a cross-layer design paradigm. While there have been extensive prior research on performance analyses of adaptive transmission techniques for non-cooperative wireless networks, majority of the literature on cooperative diversity systems are limited to a constant signalling rate and/or fixed transmit power for all nodes. More recently, the problem of optimal resource allocation in terms of power and bandwidth has been investigated for a three node cooperative wireless network [2]-[3] and for the multi-relay case in [4], although their solutions require the knowledge of CSI of all links (i.e., large overhead especially when the number of nodes in the network is large) and the source rate-adaptation was not considered. As a consequence, it is distinctly different than the adaptive source transmission policies of [5]. Motivated by these observations, [6] and [7] derived bounds for the ergodic capacity of adaptive-link cooperative relay networks with limited CSI (i.e., the destination node only needs to feedback its total received SNR to the source node) in which the rate and/or power level at the source node is adapted according to the channel condition while the relays simply amplify-and-forward or decode-and-forward their received signals to the destination node. In [8], Ikki *et. al.* followed the PDF based analytical approach in [7] to compute the ergodic capacities of an opportunistic relay-selection scheme under different source-adaptive transmission schemes. Nevertheless, all of the aforementioned analyses were limited to Rayleigh fading channels, perhaps for analytical tractability and simplicity. But it is much more realistic to model the channel gains of a practical cooperative relay network as independent and non-identically distributed (i.n.d) Rice or Nakagami-m random variables due to increased likelihood of the presence of strong specular components between the source node and the collaborating relays within a network cluster or in an airborne platform.

Inspired by the works of [9], [10], [11], [12] and [13] for simplifying and unifying the analyses of average symbol error rates and outage probability for a wide variety of digital modulation schemes with diversity receivers over generalized fading channels, we seek to develop a novel and unified analytical framework based on the MGF method for evaluating the ergodic channel capacities of dual-hop CAF relay networks under three distinct adaptive source transmission policies considered in [5] and [7]. Our approach is also motivated by the fact that the MGF of end-to-end SNR is sometimes easier to compute than its PDF and/or it may be readily available (i.e., since it is extensively used for outage probability and error rate analysis!). This development is important and of significant interest because several authors [14]-[16] have recently argued that although the MGF method has been successfully and extensively applied for evaluating the performance of wireless relaying systems in terms of outage probability and error rates, there have been very limited contributions on ergodic capacity of fading relay channels [14, pp. 2286] or explicitly highlighted the complexity of using and generalizing the MGF or the characteristic function (CHF) based approaches for channel capacity computation [15]-[16]. In fact, the lack of significant contributions on ergodic capacity analysis of cooperative relay networks can be attributed to the difficulty of evaluating the exact PDF of end-to-end SNR in closed-form. The authors of [7] and [8] circumvented this difficulty by evaluating the upper and lower bounds for the capacity instead, while [14] relied on the Jensen's inequality to derive an upper bound for the ergodic capacity over Rayleigh fading via the method of moments. More recently, Di Renzo *et. al.* [16] has proposed a general method for channel capacity analysis using a novel integral relation known as E_i –transform. But their analytical framework exhibits the following limitations:

i. The integral solutions (see [16, eq. (7) or eq. (8)] for optimal rate-adaptation with constant transmit power (ORA) policy and [16, eqs. (13)-(14) or eq. (28)] for optimal joint power-and-rate adaptation (OPRA) policy) require calculations of the derivatives of the MGF and/or its auxiliary functions, which can be very cumbersome for i.n.d fading statistics especially when the number of relays is large;

ii. Numerical results were limited only to independent and identically distributed (i.i.d) Nakagami channels perhaps due to the difficulty in computing a numerically stable MGF for the total received SNR over i.n.d Rice and/or Nakagami-Hoyt fading environments using [17, eq. (5)] along with infinite series expressions summarized in Table 1 of [17]. Even for the Nakagami-m fading environment, the final expressions for the ergodic capacities of two-hop CAF networks with ORA and OPRA policies are in the form of a double-integral and a triple-integral, respectively.

Independently, we have developed yet another unified approach for ergodic capacity analysis of cooperative relay networks over generalized fading channels which we refer to as the "cumulative distribution function (CDF) method" [12], [18]. This approach utilizes the MGF of end-to-end SNR in conjunction with an efficient multi-precision Laplace inversion formula [11], [12] to compute its CDF. Interestingly, its solution for the OPRA transmission strategy appears to be considerably simpler than that of derived in [16]. Moreover, the computational complexity for evaluating the ergodic capacities of ORA and OPRA techniques is in the form of a double-integral if the MGF of total received SNR can be specified in a closed-form.

Motivated by the above arguments and observations, the key contributions of this paper are summarized below:

i. We found an exponential-type integral representation for $\ln \gamma$, $\gamma > 0$, in which the conditional fading SNR appears only in the exponent (i.e., facilitates the averaging over fading density functions) similar to the transformation of the conditional error rate expressions as in [9]. Incidentally, our expression (A.5) reduces into [19, eq. (6)] although we show that this integral representation is valid for any $\gamma > -1$ (instead of the $\gamma > 0$ constraint given by [19, Lemma 1]). This distinction is critical since [19, eq. (6)] cannot be used for the evaluating the channel capacity with side information at both the transmitter and the receiver (as argued in [16]) without the correction.

ii. We present a novel MGF-based analytical framework for calculating the ergodic capacities of CAF relay networks with limited CSI under three distinct source-adaptive transmission policies (optimal rate-adaptation with fixed transmit power (ORA), optimal joint power and rate adaptation (OPRA), and truncated channel inversion with fixed signaling rate (TCIFR)) over i.n.d Rice and Nakagami-m fading environments. The corresponding formulas are considerably simpler and more efficient than those reported in [16] and [18].

iii. To facilitate an efficient analysis of ergodic capacity of multi-relay dual-hop CAF networks over i.n.d Rice and Nakagami-m fading channels, we derive both upper and lower bounds for the MGF of half-harmonic mean SNR in closed-form. To the best of our knowledge, ergodic capacity analyses of CAF relay networks over Rice fading environments are not available in the literature. This might be attributed to the difficulty in computing the MGF of harmonic mean SNR for each of relayed paths using the approach presented in [17] (because it involves evaluation of an integral of a product of two infinite series terms with complicated arguments). Even when a closed-form expression for the MGF bound of harmonic mean SNR (see eq. (6)) is available, it cannot be used directly in (14) and (17) or [18, eqs. (11), (15) and (19)] because we need to compute the Marcum's *Q*-function with complex arguments (which is not supported by the standard built-in functions in MATLAB and MATHEMATICA). We overcome this issue by resorting to a rapidly converging series representation of the Marcum's *Q*-function (see eq. (10)).

It is also important to highlight that our proposed MGF-based framework is appropriate for the ergodic capacity analysis of CAF relay networks with source adaptive techniques if a bound, an approximation or the exact MGF of total received SNR is available in closed-form. However, the CDF-based framework developed in [18] is more suitable if the CDF of the total received SNR is available in closed-form instead (e.g., opportunistic relay selection at the sender with selection combining at the destination node).

The remainder of this paper is organized as follows. In Section 2, the system and channel models are briefly discussed, including the development of closed-form expressions for the MGF bounds of harmonic mean SNR for dual-hop CAF multi-relay networks in Rice and Nakagami-m fading environments. The ergodic capacity analyses of CAF multi-relay networks under three distinct source adaptive transmission strategies (ORA, TCIFR and OPRA) are presented in Section 3. Selected computational results are presented in Section 4, which is followed by some concluding remarks in Section 5.

2. SYSTEM MODEL AND CHANNEL STATISTICS

Fig. 1 illustrates the link-adaptive cooperative wireless network under consideration. The source node *S* communicates with the destination node *D* via a direct-link and through *N* amplify-and-forward relays, R_i, $i \in \{1,2,....,N\}$, in two transmission phases. During the initial phase, node *S* broadcasts signal *x* to node *D* as well as to the relays R_i, where the channel fading coefficients between *S* and *D*, *S* and the i^{th} relay node R_i, and R_i and *D* are denoted by $\alpha_{s,d}$, $\alpha_{s,i}$ and $\alpha_{i,d}$, respectively. During the second phase of cooperation, each of the *N* relays transmits the received signal after amplification via orthogonal transmissions (e.g., time-division multiple access in a round-robin fashion and/or frequency division multiple access).

If the i^{th} relay amplifier gain is chosen as $G_i = \sqrt{E_s/(E_s|\alpha_{s,i}|^2 + N_0)}$ (where E_s denotes the average symbol energy and N_0 corresponds to the noise variance) and maximal-ratio combiner (MRC) is used to coherently combine all the signals during the two transmission phases, the total received SNR at the output of MRC detector is given by [1]

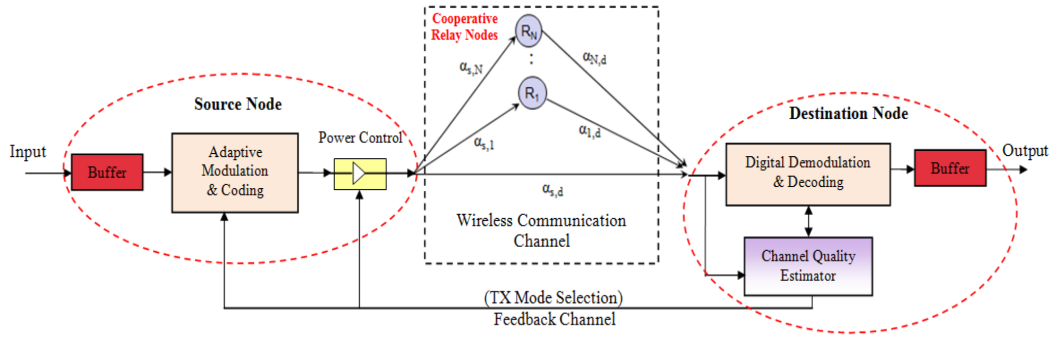

Fig. 1 Link-adaptive cooperative diversity network for ensuring the connectivity and network stability needed to support varying quality-of-service requirements in a wireless network.

$$\gamma = \gamma_{s,d} + \sum_{i=1}^{N} \frac{\gamma_{s,i}\gamma_{i,d}}{1+\gamma_{s,i}+\gamma_{i,d}} = \gamma_{s,d} + \sum_{i=1}^{N} \gamma_i \approx \gamma_{s,d} + \sum_{i=1}^{N} \gamma_i^{(TB)} \quad (1)$$

where $\gamma_i^{(TB)} = \dfrac{\gamma_{s,i}\gamma_{i,d}}{\gamma_{s,i}+\gamma_{i,d}}$ and $\gamma_{a,b} = |\alpha_{a,b}|^2 E_s/N_o$ corresponds to the instantaneous SNRs of link *a-b*. The approximation on the right-side of (1) is obtained by recognizing that the instantaneous SNR of a two-hops path can be accurately estimated to be the half harmonic mean of individual link SNRs especially at moderate and high SNR regimes [22]. Suppose $\gamma_{s,d}$, $\gamma_{s,i}$, $\gamma_{i,d}$ are i.n.d random variates, we can immediately show that the MGF of γ in (1) is given by

$$\phi_{\gamma}(s) = \phi_{\gamma_{s,d}}(s) \prod_{i=1}^{N} \phi_{\gamma_i}(s) \quad (2)$$

where $\phi_{\gamma_{s,d}}(s)$ corresponds to the MGF of SNR of the S-D link while and $\phi_{\gamma_i}(s)$ denotes the exact MGF of end-to-end SNR for a dual-hop relayed path. However, finding $\phi_{\gamma_i}(s)$ in a generalized fading environment can be a very daunting task, with existing results limited only to Rayleigh [23] and Nakagami-m [24] environments. Even in such cases, the final expressions are too complicated for further manipulations. For example, the exact MGF of SNR for a dual-hop CAF with i.n.d Nakagami-m fading statistics derived in [24] involves triple summation terms involving k^{th} derivative of a product of Whittaker functions, which is not easily evaluated using a general computing platform, besides being restrictive to positive integer *m* values. Other "exact" formulas (i.e., half-harmonic mean bound of the exact end-to-end SNR, $\gamma_i^{(TB)}$) for the MGF of SNR in a relayed path can be found in [21] (for i.n.d Rayleigh channels), [22] (for i.i.d Nakagami-m channels) and [17]. While the MGF-based approach developed in [17] is quite interesting and can be applied to a wide range of fading environments, the resulting integral expressions are often too complicated to compute and/or time-consuming (due to the need to evaluate a nested two-fold integral term with complicated arguments that might include infinite series in some cases such as Rice fading). To circumvent the aforementioned difficulties, both upper and lower bounds have been proposed and developed for $\gamma_i^{(TB)}$, viz.,

$$\gamma_i^{(LB)} = \tfrac{1}{2}\min(\gamma_{s,i},\gamma_{i,d}) \le \gamma_i^{(TB)} \le \gamma_i^{(UB)} = \min(\gamma_{s,i},\gamma_{i,d}) \quad (3)$$

Hence performance bounds can be developed by utilizing the bounds for the MGF of total received SNR using the inequality in [7] (i.e., see (3)), viz.,

$$\phi_{\gamma_{s,d}}(s)\prod_{i=1}^{N}\phi_{\gamma_i}^{(UB)}(s) \le \phi_{\gamma}(s) \le \phi_{\gamma_{s,d}}(s)\prod_{i=1}^{N}\phi_{\gamma_i}^{(LB)}(s) \quad (4)$$

where $\phi_{\gamma_i}^{(UB)}(s)$ and $\phi_{\gamma_i}^{(LB)}(s) = \phi_{\gamma_i}^{(UB)}(s/2)$ correspond to the MGFs of $\gamma_i^{(UB)}$ and $\gamma_i^{(LB)}$, respectively.

Next, we will summarize and/or derive the MGFs of $\gamma_i^{(TB)}$ and $\gamma_i^{(UB)}$ in closed-form for Rayleigh, Rice and Nakagami-m fading environments.

2.1 Rayleigh Fading Channel

The MGF of $\gamma_i^{(TB)}$ in a Rayleigh channel with i.n.d fading statistics is given by [21]

$$\phi_{\gamma_i}^{(TB)}(s) = \left[(1/\Omega_{s,i} - 1/\Omega_{i,d})^2 + (1/\Omega_{s,i} + 1/\Omega_{i,d})s\right]/\Delta^2 + \frac{2s}{\Delta^3 \Omega_{s,i}\Omega_{i,d}} \ln\left(\left(s + \Delta + \frac{1}{\Omega_{s,i}} + \frac{1}{\Omega_{i,d}}\right)^2 \frac{\Omega_{s,i}\Omega_{i,d}}{4}\right) \quad (5)$$

where $\Delta = \sqrt{(1/\Omega_{s,i} - 1/\Omega_{i,d})^2 + 2s(1/\Omega_{s,i} + 1/\Omega_{i,d}) + s^2}$, and $\Omega_{a,b} = E[\gamma_{a,b}]$ denotes the mean link SNR .

2.2 Rice Fading Channel

To the best of our knowledge, analysis and/or computational results for the ergodic capacities of adaptive-link CAF multi-relay networks over Rice fading have not been considered in the literature. This may be attributed to analytical intractability/numerical instability that arises in the computation of the MGF of total received SNR (i.e., (2)) in conjunction with the MGF of half-harmonic mean SNR for each of the relayed paths via [17, eq. (5) and Table 1]. In this case, evaluation of the MGF of $\gamma_i^{(TB)}$ for each relayed paths involves an integration of a product of two infinite series containing modified Bessel functions! Hence in the Appendix D, we derive a tractable and closed-form expression for the MGF of $\gamma_i^{(UB)}$ and $\gamma_i^{(LB)}$ over Rice fading with i.n.d fading statistics, viz.,

$$\phi_{\gamma_i}^{(UB)}(s) = \sum_{\substack{k \in \{(s,i),(i,d)\} \\ j \neq k}} 2A_k e^{-K_k} I\left[\sqrt{2A_j}, \sqrt{2K_j}, \sqrt{2A_k K_k}, 2(s + A_k)\right] \quad (6)$$

where $A_i = 1 + K_i/\Omega_i$, $I[a,b,c,d] = \frac{1}{d} e^{\frac{c^2}{2d}} Q\left(b\sqrt{\frac{d}{d+a^2}}, \frac{ac}{\sqrt{d(d+a^2)}}\right) - \frac{a^2}{d(d+a^2)} e^{\frac{c^2 - b^2 d}{2(d+a^2)}} I_0\left(\frac{abc}{d+a^2}\right)$ and the corresponding MGF for the lower bound may be computed as $\phi_{\gamma_i}^{(LB)}(s) = \phi_{\gamma_i}^{(UB)}(s/2)$.

2.3 Nakagami-m Fading Channel

A simple closed-form formula for the MGF of $\gamma_i^{(TB)}$ over Nakagami-m fading environment with i.i.d fading statistics has been derived in [22], viz.,

$$\phi_{\gamma_i}^{(TB)}(s) = {}_2F_1\left(m, 2m; m + 0.5; -s\Omega/4m\right) \quad (7)$$

where *m* denotes the Nakagami-m fading severity index. To facilitate performance analyses in a more realistic operating environment with i.n.d fading statistics among the spatially distributed relay nodes, we also derive a closed-form formula for the MGF of $\gamma_i^{(UB)}$ in the Appendix E for a dual-hop CAF relayed path, viz.,

$$\phi_{\gamma_i}^{(UB)}(s) = \sum_{\substack{k \in \{(s,i),(i,d)\} \\ j \neq k}} \frac{\Gamma(m_k + m_j)}{m_k \Gamma(m_k)\Gamma(m_j)} \left(\frac{\Omega_j m_k}{s\Omega_j \Omega_k + \chi_{j,k}}\right)^{m_k} {}_2F_1\left(1 - m_j, m_k;\ 1 + m_k; \frac{(s\Omega_k + m_k)\Omega_j}{s\Omega_j \Omega_k + \chi_{j,k}}\right) \quad (8)$$

where $\chi_{j,k} = \Omega_j m_k + \Omega_k m_j$. It is also important to highlight that the Gauss hypergeometric function in (8) will reduce into a finite series if $m_{s,i}$ and $m_{i,d}$ are positive integers.

2.4 Outage Probability

Outage probability is defined as the probability that the total received SNR γ (as defined in (1)) falls below a specified threshold value γ_0, i.e., $P_{\text{out}} = F_\gamma(\gamma_0)$. The knowledge of P_{out} or equivalently, the CDF of total received SNR is required in the evaluation of OPRA and TCIFR capacities of CAF relay networks. In general, analytical derivation of the CDF of γ is not necessarily trivial. However, this quantity can be evaluated numerically and efficiently with the aid of Abate's multi-precision Laplace inversion formula [12] once the MGF of γ is found, viz.,

$$F_\gamma(x) \cong \frac{1}{2Z}\phi_\gamma(r)e^{rx} + \frac{r}{Z}\sum_{k=1}^{Z-1} \mathrm{Re}\{\frac{1+j\sigma(\theta_k)}{s(\theta_k)} e^{xs(\theta_k)}\phi_\gamma\left(s(\theta_k)\right)\} \tag{9}$$

where $r = 2Z/(5x)$, $\theta_k = k\pi/Z$, $\sigma(\theta_k) = \theta_k + (\theta_k \cot(\theta_k) - 1)\cot(\theta_k)$, $s(\theta_k) = r\theta_k(j + \cot(\theta_k))$ and the positive integer Z in (9) can be chosen appropriately to achieve the desired accuracy. It should be emphasized that the above is not the only recommended solution but rather one may also resort to other numerical Laplace inversion techniques such as the Euler summation approach in [11] or via the saddle-point approximation.

Nevertheless, computations of outage probability bounds for CAF multi-relay networks over Rice fading channels using (6) in (9) or [11] pose some difficulties (since they require complex arguments to be passed into the MGF formula (6) that contains the Marcum *Q*-function). Unfortunately, the built-in function for evaluating the Marcum *Q*-function on commercial mathematical software packages such as MATLAB does not support complex arguments. To overcome this minor snag, we wrote a new MATLAB routine for computing the generalized Marcum *Q*-function (that can handle complex arguments) based on its rapidly convergent canonical series representation given by [25, eq. (7)],

$$Q_M(\sqrt{2a}, \sqrt{b}) = 1 - \sum_{k=0}^{\infty} \frac{a^k e^{-a}}{k!} \frac{G(M+k, \frac{b}{2})}{\Gamma(M+k)} \tag{10}$$

where $G(a,x) = \int_0^x t^{a-1}e^{-t}dt$ denotes the lower incomplete Gamma function. Although the saddle point approximation circumvents the requirement for $Q(.,.)$ to handle complex arguments, this method is not very attractive for our application because it requires the first two derivatives of cumulant generating function (CGF) $K_\gamma(s) = \ln(\phi_\gamma(s))$ and also the solution to a root-finding problem that involves first-order derivative of the nonlinear CGF expression.

3. ERGODIC CAPACITY COMPUTATION IN FADING CHANNELS

Ergodic capacity is an important and a basic tool for appraisal, design and optimization of new wireless communication techniques because this metric yields an information-theoretic bound on the achievable average rate for reliable communication over fading channels. It can also be used to gain insights into how and to what degree opportunistic/adaptive transmission strategies (realized by exploiting CSI that may be made available at the transmitter, or the receiver, or both) and diversity schemes can counteract the detrimental effects of fading. Thus in this section, we derive unified analytical expressions (based on a novel MGF method that is different and considerably simpler than that of [16]) for evaluating the ergodic capacities of CAF relay networks in conjunction with three adaptive source transmission techniques over a myriad of fading environments.

3.1 Optimal Rate Adaptation with a Fixed Transmit Power (ORA)

The ergodic capacity of a multi-relay CAF network (with limited CSI feedback) when only the rate at the transmitter is dynamically adapted to the time-varying channel conditions and constant transmit power constraint is given by [7]

$$\frac{\bar{C}_{ORA}}{B} = \frac{1}{N+1}\frac{1}{\ln 2}\int_0^\infty \ln(1+\gamma) f_\gamma(\gamma)d\gamma \tag{11}$$

where B and N denote the channel bandwidth and number of cooperating relays, respectively. This corresponds to fading channel capacity with side information at the receiver only [5].

Utilizing an "exponential-type" integral representation for $\ln(\gamma+1)$ (please refer to Appendix A for details), we can readily facilitate the averaging problem in (11) given that $\phi_\gamma(\cdot)$ is available in closed-form. Substituting (A.5) or [19, eq. (6)] into (11), we obtain the ergodic capacity in terms of the MGF of γ alone, viz.,

$$\begin{aligned}\frac{\bar{C}_{ORA}}{B} &= \frac{1}{N+1}\frac{1}{\ln 2}\int_0^\infty \frac{e^{-x}}{x}\left[\int_0^\infty (1-e^{-x\gamma}) f_\gamma(\gamma) d\gamma\right] dx \\ &= \frac{1}{N+1}\frac{1}{\ln 2}\int_0^\infty \frac{e^{-x}}{x}\left[1-\phi_\gamma(x)\right] dx\end{aligned} \tag{12}$$

It is also interesting to note that (12) allows us to prove that the ergodic capacity for point-toto-point communication systems increases with the increasing receiver-diversity order regardless of the fading channel model or diversity combining technique employed. This is because the term $1-\phi_\gamma(x)$ in (12) approaches to much closer to unity as receiver-diversity order increases (since $0 \le \phi_\gamma(s) \le 1$ is a monotonically decreasing function with respect to its argument). Gunther [26, pp. 401] suggested that while this is intuitive, it is not easy to prove this trend mathematically for the ORA policy.

3.2 Truncated Channel Inversion with Fixed Rate (TCIFR)

The ORA capacity in Section 3.1 can be achieved using a fixed-power variable-rate coding strategy. However, some real-time applications cannot tolerate the variable delays exhibited by this coding strategy. For these applications, the transmitter may use a fixed-rate coding and adapt its power to keep the total received SNR constant (such that the "channel fading is inverted"). The zero-outage capacity (also known as delay-limited capacity) is given by (13) when the cut-off SNR γ_0 is set to zero. This technique is the least complex to implement given that reliable CSI estimates are available at the transmitter and receiver [5].

However when the channel experiences deep fades, the penalty in transmit power requirement with the channel inversion strategy will be enormous because channel inversion needs to compensate for the deep fades. To address this practical implementation issue, a modified channel inversion policy was also considered in [5]. In the TCIFR technique, channel is only inverted when the received SNR γ is above a certain cut-off fade depth γ_0. In this case, it is easy to show that the outage probability is given by $P_{\text{out}} = F_\gamma(\gamma_0)$ (since data transmission is ceased when $\gamma < \gamma_0$) and the corresponding ergodic capacity as

$$\frac{\bar{C}_{TCIFR}}{B} = \frac{1}{N+1}\log_2\left(1+\frac{1}{\int_{\gamma_0}^\infty \gamma^{-1} f_\gamma(\gamma) d\gamma}\right) F_\gamma^c(\gamma_0) \tag{13}$$

where $F_\gamma^c(x) = 1 - F_\gamma(x)$ denotes the complementary CDF of γ.

Substituting (B.2) (i.e., the PDF of γ is expressed as an inverse Fourier transform integral of its CHF) into (13), we obtain (after some manipulations as discussed in Appendix B)

$$\frac{\bar{C}_{TCIFR}}{B} = \frac{1}{N+1}\log_2\left(1-\frac{1}{\nabla}\right) F_\gamma^c(\gamma_0) \tag{14}$$

where $\nabla = \frac{1}{\pi}\int_0^\infty \mathrm{Re}\{\phi_\gamma(-j\omega) E_i(-j\omega\gamma_0)\} d\omega$ and the exponential integral with an imaginary argument $E_i(-jc)$ may be evaluated in MATLAB as "$\cos\mathrm{int}(c) - j(-\pi/2 + \sin\mathrm{int}(c))$". Moreover, the cut-off SNR γ_0 in (13) can be chosen to meet a specified P_{out} or to maximize $\bar{C}_{TCIFR}/B$.

3.3 Optimal Joint Power-and-Rate Adaptation (OPRA)

Under an average transmit power constraint, the OPRA strategy seeks to dynamically adapt both the transmission power and rate relative to the channel quality through use of a multiplexed multiple codebook design [5]. This leads to the highest achievable capacity with perfect CSI at the transmitter and the receiver. In [29], the authors' have advocated a simpler approach (from the implementation stand-point) to achieve this capacity using a single codebook (i.e., fixed rate) with variable power transmission because the former is inherently hard to implement. From [5], we have

$$\frac{\overline{C}_{OPRA}}{B}=\frac{1}{N+1}\frac{1}{\ln 2}\int_{\gamma_0}^{\infty}\ln\left(\gamma/\gamma_0\right)f_\gamma(\gamma)d\gamma \tag{15}$$

where γ_0 is the optimal cut-off SNR below which the data transmission is suspended. Substituting (A.4) into (15), and changing the order of integration, we get

$$\begin{aligned}\frac{\overline{C}_{OPRA}}{B}&=\frac{1}{N+1}\frac{1}{\ln 2}\int_0^{\infty}\frac{1}{x}\left[e^{-2x}\int_{\gamma_0}^{\infty}f_\gamma(\gamma)d\gamma-\int_{\gamma_0}^{\infty}e^{\frac{-2x\gamma}{\gamma_0}}f_\gamma(\gamma)d\gamma\right]dx\\&=\frac{1}{N+1}\frac{1}{\ln 2}\int_0^{\infty}\frac{1}{x}\left[e^{-2x}[1-F_\gamma(\gamma_0)]-\phi_\gamma(2x/\gamma_0,\gamma_0)\right]dx\end{aligned} \tag{16}$$

where $\int_{\gamma_0}^{\infty}f_\gamma(\gamma)d\gamma=1-F_\gamma(\gamma_0)=F_\gamma^c(\gamma_0)$ and $\int_{\gamma_0}^{\infty}e^{-s\gamma}f_\gamma(\gamma)d\gamma=\phi_\gamma(s,\gamma_0)$ denotes the marginal MGF of random variable γ. Since the desired marginal MGF is typically not available in closed-form, we circumvent this difficulty by computing this term as the "CDF" of an auxiliary function using (9) and (C.2) (additional details are provided in Appendix C), viz.,

$$\frac{\overline{C}_{OPRA}}{B}=\frac{1}{N+1}\frac{1}{\ln 2}\int_0^{\infty}\frac{1}{x}\left[e^{-2x}[1-F_\gamma(\gamma_0)]-\phi_\gamma\left(2x/\gamma_0\right)+F_{\hat{\gamma}}(\gamma_0)\right]dx \tag{17}$$

where the term $F_{\hat{\gamma}}(\gamma_0)$ can be evaluated using (9) or [11] but with $\phi_\gamma(s+2x/\gamma_0)$ instead of $\phi_\gamma(s)$. Therefore it is evident that if $\phi_\gamma(s)$ is available in closed-form, then the integrand in (17) can be evaluated very efficiently using (9), and computational complexity of (17) is no more complicated that of a double integral.

To achieve the capacity (17), the channel fade level (i.e., CSI) tracked at the receiver must be conveyed to the transmitter on the feedback path for dynamic power and rate adaptation. Since data transmission is suspended when $\gamma<\gamma_0$, this optimal adaptation policy suffers a probability of outage given by $P_{\text{out}}=F_\gamma(\gamma_0)$, which equals to the probability of no transmission. Moreover, the optimal cut-off SNR γ_0 must satisfy

$$F_\gamma^c(\gamma_0)-\gamma_0[1+\int_{\gamma_0}^{\infty}\gamma^{-1}f_\gamma(\gamma)d\gamma]=0 \tag{18}$$

The integral term in (18) can be evaluated very efficiently by following the development of (14) or (B.3), especially when the MGF of γ is available in closed-form. Hence we can find the optimal cut-off SNR γ_0 by solving the equation $F_\gamma^c(\gamma_0)=\gamma_0(1+\nabla)$ for γ_0 numerically. Furthermore, asymptotic analysis of (18) shows that $\gamma_0\to 0$ when the mean SNR $\Omega\to 0$ because $F_\gamma(\gamma)\to 1$ and $f_\gamma(\gamma)\to 0$ (i.e., the effect of $\Omega\to 0$ can be predicted from the normalized PDF and CDF curves when its argument $\gamma\to\infty$). Similarly, we observe that $\gamma_0\to 1$ as $\Omega\to\infty$ because $F_\gamma(\gamma)\to 0$ and $\phi_\gamma(s)\to 0$. Therefore, the optimal cut-off SNR γ_0 always lies in the interval [0, 1] regardless of the assumptions on fading channel models, diversity techniques used an/or the number of cooperating relay nodes.

Before concluding this section, we would like to highlight that our final ergodic capacity formulas in (12), (14) and (17) for ORA, TCIFR and OPRA schemes, respectively, are all expressed in terms of only the MGF and/or the CDF of the total received SNR. Hence we can efficiently compute the appropriate bounds or tight approximations for the ergodic capacity of CAF relay networks under different source adaptive transmission techniques and fading environments by utilizing the closed-form MGF formulas for the exact half-harmonic mean SNR (i.e., (5) and (7)) or bounds for the half-harmonic mean SNR (i.e., (6) and (8)) in conjunction with (2) and (9). It is evident that our formulas are considerably simpler and more efficient than the MGF-approach based on the E_i-transform proposed in [16].

4. NUMERICAL RESULTS

In this section, selected numerical results are presented for the ergodic capacities of CAF relay networks with different source-adaptive transmission techniques over Rayleigh, Nakagami-m and Rice fading environments. One of the objectives here is to investigate the accuracy, reliability and numerical stability of our proposed analytical framework. The following average link SNRs (arbitrarily chosen) will be used to generate the plots in this paper, unless stated otherwise: $\Omega_{s,1} = \Omega_{2,d} = E_s/N_o$, $\Omega_{s,2} = \Omega_{1,d} = 0.5E_s/N_o$, and $\Omega_{s,d} = 0.2E_s/N_o$.

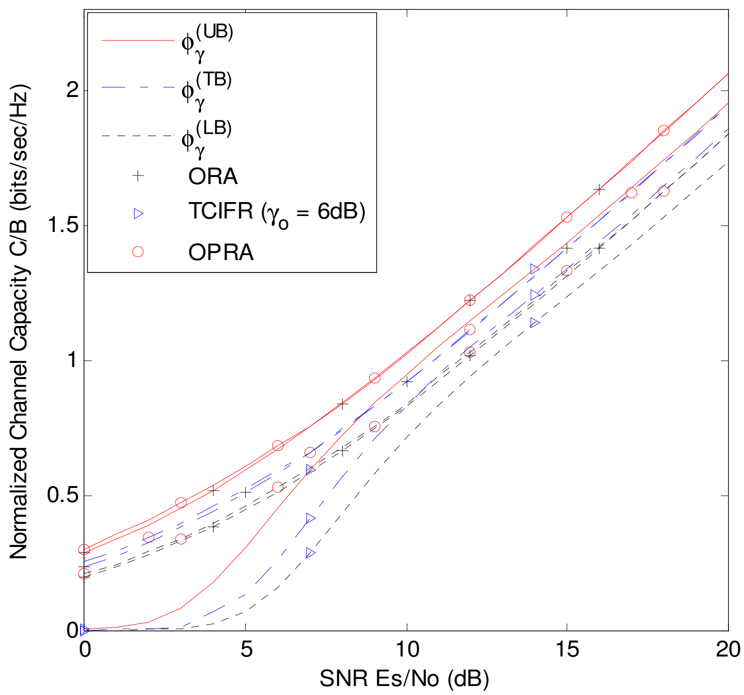

Fig. 2 Ergodic capacities of a cooperative relay network with ORA, OPRA and TCIFR policies in an i.n.d Rayleigh fading channel with two cooperative relays (N = 2).

Fig. 2 depicts a comparison between the ergodic channel capacities of three source adaptive transmission policies in an i.n.d Rayleigh fading channels with two cooperative relays. The curves corresponding to the upper and lower bounds were obtained using (4) and (8) by setting m = 1 (or using (6) by setting K = 0) while the curves corresponding the “tight-bound” case is obtained using (2) in conjunction with (5). It is apparent that the ergodic capacity with both transmitter and receiver CSI is only negligibly larger than the capacity with receiver CSI only (i.e., since there is no significant difference observed in the capacities of OPRA and ORA at high SNR). But the ergodic capacity of TCIFR policy (plotted for the cut-off SNR γ_0 = 6 dB) is considerably lower than the OPRA and ORA schemes. Although not shown in this figure, we also noticed that the curves corresponding to the "tight bound" are in good agreement with the Monte Carlo simulation results that corresponds to the "exact" ergodic capacity). Moreover, the "exact" ergodic capacity is slightly closer to the lower bound (rather than the upper bound) especially at lower values of E_s/N_o. Although [7] has studied the channel capacities of cooperative relaying system in an i.n.d Rayleigh fading channel, but their framework does not lend itself to the analysis of the "tight bound" case or generalize to any other fading channels, whereas our framework encapsulates all these cases in a unified way (e.g., see Fig. 4 for i.i.d Nakagami-m fading, Fig. 5 for i.n.d Rice fading and Fig. 7 for i.n.d Nakagami-m fading). This in turn demonstrates the generality and utility of our proposed analytical framework, even for the specific instance of ergodic capacity analysis in Rayleigh fading.

In Fig. 3, the ergodic capacity of a CAF relay network with TCIFR policy is plotted as a function of the cut-off SNR at E_s/N_o = 6 dB and E_s/N_o = 15 dB. It is evident that there exists an optimal choice for the cut-off SNR which maximizes the ergodic channel capacity when E_s/N_0 is fixed. But it should be also emphasized that the selection of γ_0 will directly affect the outage probability performance (i.e., probability of no transmission).

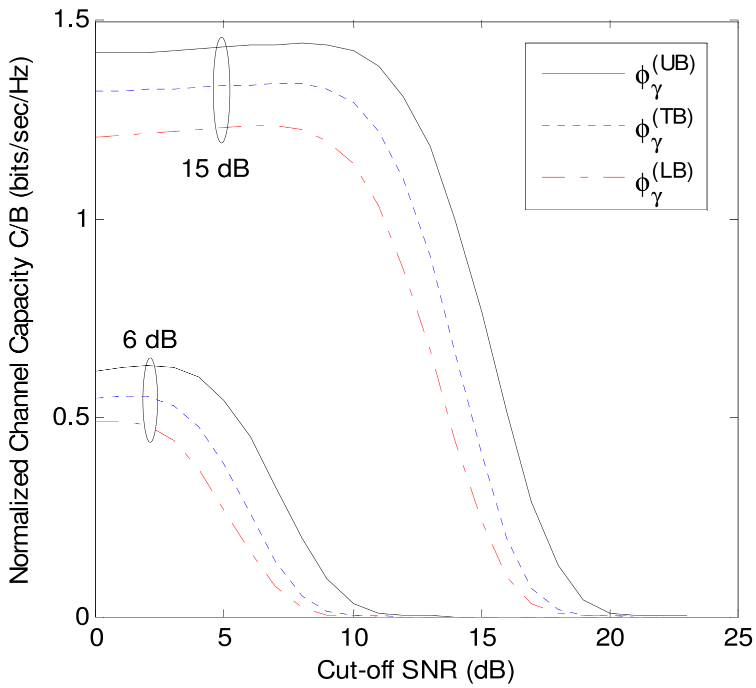

Fig. 3 Ergodic capacity of a CAF relay network with two cooperating relays and TCIFR policy plotted as a function of cut-off SNR γ_0 in an i.n.d Rayleigh fading environment.

Fig. 4 shows the ergodic capacities of a classical 3-node CAF relay network (i.e., N = 1) with different source-adaptive transmission schemes over i.i.d Nakagami-m channels (for fading severity indices m = 0.5, 1, 1.5 and 2). These performance curves were generated using (2) in conjunction with (7). It should clear by now that our framework does not impose any restrictions of the fading severity index m (i.e., can handle non-integer m values). We observe that the ergodic capacity increases with the increasing value of m for all source adaptive transmission schemes, as anticipated (i.e., ergodic capacity is higher when the channel experiences less severe fading). We also noticed that the gap between the curves corresponding to ORA and OPRA schemes widens as m decreases, although their maximum achievable average transmission rates are quite similar at higher values of E_s/N_o.

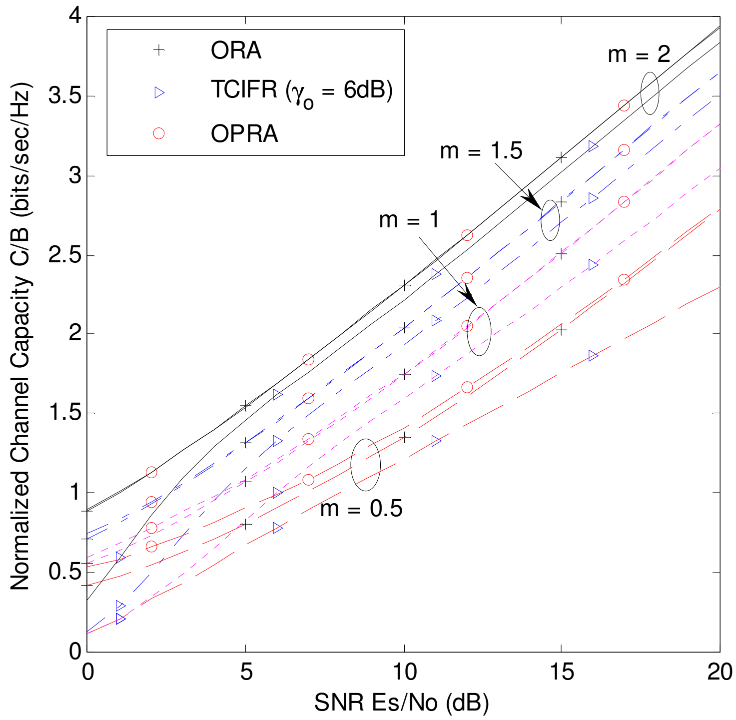

Fig. 4 Ergodic capacities of ORA, OPRA and TCIFR policies over i.i.d Nakagami-m fading environments with a single CAF relay (N = 1).

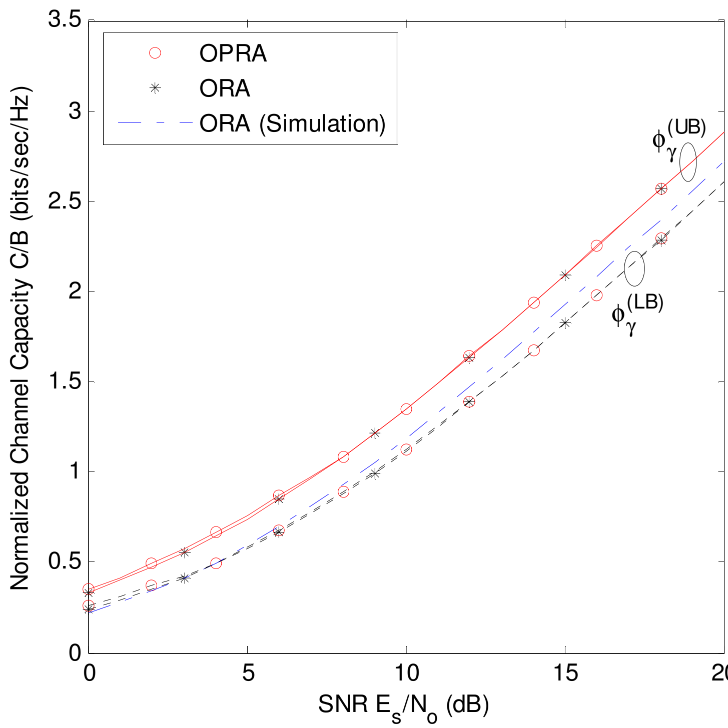

Fig. 5 Ergodic capacities of a CAF relay network (N = 1) with ORA and OPRA schemes over an i.n.d Rice fading environment (Rice factor K = 3).

Fig. 5 illustrates the ergodic capacity comparison between OPRA and ORA schemes in a 3-node CAF relay network ($N = 1$) over an i.n.d Rice fading environment ($K = 3$). To generate the curves, we have utilized (4) in conjunction with (6). The general trends observed in Fig. 2 are also observed in Fig. 3. The ergodic capacity with OPRA strategy slightly outperforms the ergodic capacity with ORA scheme at the low SNR regime, but their difference diminishes as average link SNR increases. Owing to the difficulty in deriving a tractable analytical expression for the MGF of half-harmonic mean SNR in Rice fading, we validate the tightness of our upper and lower bounds via Monte Carlo simulation. Once again, we observe that the "exact" ergodic capacity (obtained via simulation) is much closer to the lower bound than its upper bound. This example also highlights the utility of our newly derived MGF bound in (6) for the design and appraisal of CAF relay networks over i.n.d Rice fading channels.

In Fig. 6, we investigate the effect of fade distribution on the ergodic capacity of a 4-node CAF relay network in i.n.d Rice fading channels. All the curves were generated by evaluating (12) in conjunction with (2) and (6). The following Rice factors are assumed (arbitrarily chosen) for the spatially-distributed wireless links in a CAF relay network: Case 1: $K_{s,1} = 2$, $K_{1,d} = 4$, $K_{s,2} = 3$, $K_{2,d} = 3$, $K_{s,d} = 1$; Case 2: $K_{s,1} = 1.5$, $K_{1,d} = 3.5$, $K_{s,2} = 2.5$, $K_{2,d} = 2.5$, $K_{s,d} = 1$; Case 3: $K_{s,1} = 2$, $K_{1,d} = 2$, $K_{s,2} = 2$, $K_{2,d} = 2$, $K_{s,d} = 1$; Case 4: $K_{s,1} = 0$, $K_{1,d} = 0$, $K_{s,2} = 0$, $K_{2,d} = 0$, $K_{s,d} = 0$. We have verified that the curve corresponding to the Case 4 matches exactly with the results in [7] (i.e., Rayleigh fading environment). Moreover, we observe that the ergodic capacity can vary considerably (due to the amount of fading experienced on each wireless link) even when the mean link SNRs are kept constant. This observation in turn indicates that the optimal transmit power allocation strategy among nodes in cooperation (i.e., that maximizes the ergodic capacity) should also take into consideration the amount of fading experienced on different wireless links, in addition to compensating for the disparity in their mean link SNRs.

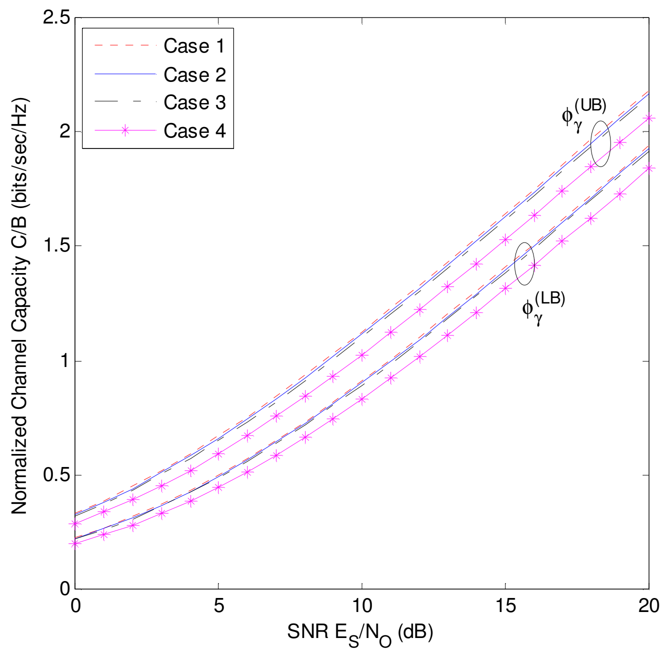

Fig. 6 Effect of the fade distributions on the ergodic capacity of a 4-node CAF relay network ($N = 2$) with ORA policy over i.n.d Rice fading channels.

Fig. 7 investigates the effect of increasing CAF diversity order on the ergodic capacity (with CSI at the receiver only) in an i.n.d Nakagami-m fading environment. The following channel parameters (arbitrarily chosen) have been considered for generating this plot: $m_{s,1} = 2$, $m_{1,d} = 2$,

$m_{s,2} = 5$, $m_{2,d} = 5$, $m_{s,d} = 1$, $\Omega_{s,d} = 0.2E_s/N_o$, $\Omega_{1,d} = 0.5E_s/N_o$ and $\Omega_{s,1} = \Omega_{s,2} = \Omega_{2,d} = E_s/N_o$. It is apparent that the ergodic capacity of CAF relay system is considerably higher than the non-cooperative diversity system (N = 0) as E_s/N_o gets smaller. However, the opposite trend is observed at larger values of E_s/N_o. This observation can be explained by noting that although CAF relaying protocol can exploit the inherent spatial diversity in wireless broadcast transmissions (which is most beneficial in poor channel conditions such as at the tactical edge or cell boundaries), there is a loss in spectral efficiency due to its inherent half-duplex operation. In fact, there is no incentive for using a cooperative diversity scheme when the source-destination link is good. This observation in turn suggests that we should adapt N based on the channel quality (i.e., increasing N as the channel condition deteriorates to provide additional diversity and maximize the capacity, while decreasing N when better channel conditions prevail).

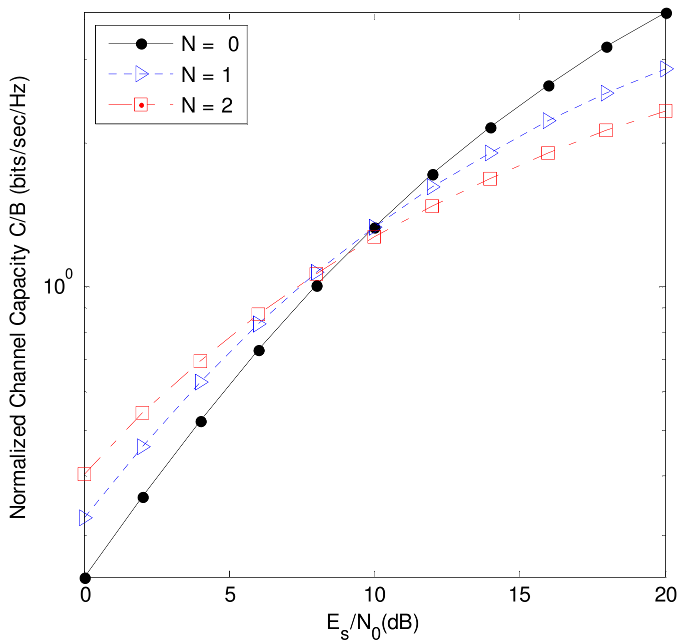

Fig.7 Effect of increasing the number of cooperating relay nodes on the ergodic capacity (upper bound) of a CAF relay network over an i.n.d Nakagami-m fading channel.

5. CONCLUSIONS

Ergodic channel capacity is an important tool for the design and appraisal of new wireless communication techniques devised to improve the spectral efficiency and/or counteract the detrimental effects of multipath fading (e.g., adaptive and opportunistic communication methods). In this article, we develop a novel MGF-based analytical framework that exploits an integral representation of the logarithmic function $\ln \gamma$, $\gamma > 0$, in a "desirable exponential-form" to unify the evaluation of ergodic channel capacities for both cooperative and non-cooperative diversity networks in conjunction with two distinct adaptive source transmission policies (i.e., ORA and OPRA) over a myriad of fading environments. The proposed method (in conjunction with the upper and lower bounds for the MGF of half-harmonic mean SNR in closed-form) also dramatically simplify the ergodic channel capacity calculations for dual-hop CAF multi-relay networks with three distinct adaptive source transmission techniques over Nakagami-m and Rice channels, and facilitates the investigation on the impact of fade distributions and/or dissimilar fading statistics across the spatially distributed communication links on the channel capacity, without imposing any restrictions on the fading parameters. Our analytical framework can be easily extended to characterize the ergodic capacities of other types of cooperative communication techniques including, but not limited to decode-and-forward and opportunistic relay selection strategies, provided the MGF of end-to-end SNR can be found.

APPENDIX A

In this appendix, we outline the derivation of an "exponential-type" integral representation for the logarithmic function ln γ when γ > 0. Such a representation will facilitate the averaging problem that is typically encountered in the capacity analysis over fading channels, and therefore leads to a unified approach for calculating the ergodic capacity of CAF relay networks in a myriad of fading environments. Utilizing [20, eq. (1.512.2)], we have

$$\ln\gamma = 2\sum_{\substack{k=1\\k\ odd}}^{\infty}\frac{1}{k}\left(\frac{\gamma-1}{\gamma+1}\right)^k = 2\sum_{\substack{k=1\\k\ odd}}^{\infty}\frac{1}{k}\left(y\right)^k, \quad \gamma>0 \tag{A.1}$$

where $y=\dfrac{\gamma-1}{\gamma+1}$. Substituting $y^k = \dfrac{1}{\Gamma(k)}\int_0^\infty x^{k-1}e^{-x/y}\,dx$ [20, eq. (3.381.4)] into (A.1), we obtain

$$\ln\gamma = 2\sum_{\substack{k=1\\k\ odd}}^{\infty}\frac{1}{k}\left(\frac{1}{\Gamma(k)}\int_0^\infty x^{k-1}e^{-x/y}\right)dx = 2\int_0^\infty e^{-x/y}\left(\sum_{\substack{k=1\\k\ odd}}^{\infty}\frac{1}{k!}x^{k-1}\right)dx \tag{A.2}$$

Recognizing that $\dfrac{1}{x}\ \mathrm{s}h\ x = \dfrac{e^x-e^{-x}}{x} = \sum_{\substack{k=1\\k\ odd}}^{\infty}\dfrac{1}{k!}x^{k-1}$ [20, Eq. (1.411.2)], (A.2) can be re-stated as

$$\ln\gamma = 2\int_0^\infty \frac{1}{x}(e^x-e^{-x})\,e^{-x\left(\frac{\gamma+1}{\gamma-1}\right)}dx \tag{A.3}$$

Finally using variable substitution $x = z(\gamma-1)$, $dz = \dfrac{dx}{\gamma-1}$, we arrive at (A.4) after some routine algebraic manipulations, viz.,

$$\ln\gamma = \int_0^\infty \frac{1}{z}\left[\ e^{-2z}-e^{-2z\gamma}\right]dz, \quad \gamma>0 \tag{A.4}$$

It is also obvious from (A.4) that

$$\ln(1+\gamma) = \int_0^\infty \frac{e^{-2z}}{z}\left[\ 1-e^{-2z\gamma}\right]dz = \int_0^\infty \frac{e^{-x}}{x}\left[\ 1-e^{-x\gamma}\right]dx, \quad \gamma>-1 \tag{A.5}$$

Incidentally, the second term in (A.5) is identical to [19, eq. (6)]. However, it should be emphasized that in Lemma 1 of [19, eq. (6)], the author indicated that his representation is valid only for γ > 0 instead of γ > -1 (from our derivation). In fact, if one start the derivation with the power series for ln γ shown in [20, eq. (1.512.3)] (which was used in [19]), then the representation in (A.4) would be valid for any $\gamma \geq 0.5$. In this case, the resulting expression [19, eq. (6)] cannot be used for the ergodic capacity analysis of the OPRA policy. Perhaps for this reason, [16] abandoned the approach in [19], and try to develop yet another MGF method based on the Ei-transform to unify the analysis of ergodic capacity over generalized fading channels.

APPENDIX B

Let $\phi_X(s) = \int_0^\infty e^{-sx} f_X(x)dx$ and $\Phi_X(j\omega) = \int_0^\infty e^{j\omega x} f_X(x)dx$ denote the MGF and the characteristic function (CHF) of random variable $X \geq 0$, respectively. In this case, the CHF is related to the MGF as $\Phi_X(j\omega) = \phi_X(-j\omega)$ and the probability density function (PDF) of *X* (may be expressed as an inverse Fourier transform of its CHF) is given by

$$f_X(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\phi_x(-j\omega)e^{-j\omega x}d\omega \tag{B.1}$$

If we express the CHF of random variable X in its polar form $\Phi_x(j\omega)=|\Phi_x(j\omega)|e^{j\theta(\omega)}$, then (B.1) may be re-stated as

$$\begin{aligned}f_X(x)&=\frac{1}{2\pi}\int_{0}^{\infty}\Phi_X(j\omega)e^{-j\omega x}d\omega+\frac{1}{2\pi}\int_{-\infty}^{0}\Phi_X(j\omega)e^{-j\omega x}d\omega\\&=\frac{1}{\pi}\int_0^{\infty}|\Phi_X(j\omega)|\cos(\theta(\omega)-\omega x)\,d\omega\\&=\frac{1}{\pi}\int_0^{\infty}\mathrm{Re}\{\Phi_X(j\omega)e^{-j\omega x}\}\,d\omega\end{aligned} \tag{B.2}$$

Consequently, we can simplify an integral that arises in the ergodic capacity analysis of OPRA and TCIFR techniques as

$$\int_1^{\infty}\gamma^{-1}f_\gamma(\gamma_0\gamma)d\gamma=\frac{1}{\pi}\int_0^{\infty}\mathrm{Re}\left\{\Phi_\gamma(j\omega)\left(\int_1^{\infty}\frac{e^{-j\omega\gamma_0\gamma}}{\gamma}d\gamma\right)\right\}d\omega=-\frac{1}{\pi}\int_0^{\infty}\mathrm{Re}\left\{\phi_\gamma(-j\omega)E_i(-j\omega\gamma_0)\right\}d\omega \tag{B.3}$$

with the aid of (B.2) and recognizing that $Ei(-q)=-\int_q^{\infty}\frac{e^{-t}}{t}dt=-\int_1^{\infty}\frac{e^{-qt}}{t}dt$. Although the exponential integral $E_i(x)$ is usually defined for real $x < 0$, it is quite straight-forward to show that this function is also well-defined if its argument is purely imaginary. This is particularly interesting in that our unified expressions for the ergodic capacity with TCIFR policy and the transcendental equation for computing the optimal cut-off SNR γ_0 for the OPRA policy can expressed in terms of $E_i(-jc)$ where $c>0$ is real. Utilizing the Euler identity, we can express $E_i(-jc)$ in terms of the familiar cosine-integral and sine-integrals, viz.,

$$E_i(\mp jc)=-\int_1^{\infty}\frac{\cos(ct)}{t}\pm j\int_1^{\infty}\frac{\sin(ct)}{t}=\mathrm{ci}(c)\mp j\mathrm{si}(c) \tag{B.4}$$

with the aid of [20, eq. (3.721.2) and eq. (3.721.3)]. Hence, $E_i(-jc)$ can be evaluated in MATLAB using the command line "$\cos\mathrm{int}(c)-j(-\pi/2+\sin\mathrm{int}(c))$".

APPENDIX C

It is important to note that the knowledge of the marginal MGF of end-to-end SNR may be required while evaluating the ergodic capacity with OPRA policy (e.g., see (16)). However, this quantity is generally not available in closed-form. But if a closed-form expression for the MGF $\phi_\gamma(\cdot)$ is available, we may then use (9) (i.e., multi-precision Laplace inversion formula [12]) or [11] for computing the desired marginal MGF very efficiently as illustrated below. A similar technique was also considered in [16] for computing the truncated MGF of SNR.

Let us define an auxiliary function $f_{\hat{\gamma}}(x)=\exp(-\beta x)f_\gamma(x)$. Hence the marginal MGF of the total received SNR can be evaluated as

$$\begin{aligned}\phi_\gamma(\beta,\alpha)&=\int_\alpha^{\infty}e^{-\beta\gamma}f_\gamma(\gamma)d\gamma=\int_\alpha^{\infty}f_{\hat{\gamma}}(\gamma)d\gamma=F_{\hat{\gamma}}(\infty)-F_{\hat{\gamma}}(\alpha)\\&=\phi_\gamma(\beta)-F_{\hat{\gamma}}(\alpha)\end{aligned} \tag{C.1}$$

where $F_{\hat{\gamma}}(y)=\int_0^y f_{\hat{\gamma}}(x)dx$. It is obvious that $F_{\hat{\gamma}}(\infty)=\int_0^{\infty}e^{-\beta x}f_\gamma(x)dx=\phi_\gamma(\beta)$ and the "MGF" of the auxiliary function can be also expressed in closed-form, viz.,

$$\phi_{\hat{\gamma}}(s) = \int_0^\infty e^{-sx} f_{\hat{\gamma}}(x)dx = \int_0^\infty e^{-(s+\beta)x} f_\gamma(x)dx = \phi_\gamma(s+\beta) \tag{C.2}$$

Therefore we can evaluate the second term $F_{\hat{\gamma}}(\alpha)$ in (C.1) and/or the desired marginal MGF efficiently using Abate's fixed-Talbot method (9) or [11] in conjunction with (C.2).

APPENDIX D

In this appendix, we derive closed-form expressions for the upper and lower MGF bounds for the half harmonic mean SNR (i.e., relayed path of a dual-hop CAF network) over i.n.d Rice fading environments. The PDF of $\gamma_i^{(UB)} = \min(\gamma_{s,i}, \gamma_{i,d})$ for a 2-hop relayed path is given by

$$f_{\gamma_i}^{(UB)}(x) = f_{\gamma_{s,i}}(x)\left[1-F_{\gamma_{i,d}}(x)\right] + f_{\gamma_{i,d}}(x)\left[1-F_{\gamma_{s,i}}(x)\right] = \sum_{\substack{k \in \{(s,i),(i,d)\} \\ j \neq k}} f_{\gamma_k}(x)\left[1-F_{\gamma_j}(x)\right] \tag{D.1}$$

where $f_{\gamma_q}(x)$ and $F_{\gamma_q}(x)$ correspond to the PDF and CDF of fading SNR γ_q respectively, which for the Rice channel are given by [27, pp. 349]

$$f_{\gamma_q}(x) = \left(\frac{1+K_q}{\Omega_q} e^{-K_q}\right) e^{-x(1+K_q)/\Omega_q} I_0\left(2\sqrt{\frac{K_q(1+K_q)x}{\Omega_q}}\right), \quad x \geq 0 \tag{D.2}$$

$$F_{\gamma_q}(x) = 1 - Q\left(\sqrt{2K_q}, \sqrt{\frac{2(1+K_q)x}{\Omega_q}}\right) \tag{D.3}$$

where K_q denotes the Rice fading parameter, $\Omega_q = E[\gamma_q]$ corresponds to the mean link SNR, $I_0(.)$ is the zero-order modified Bessel function and $Q(.,.)$ is the first-order Marcum Q-function. Substituting, (D.2) and (D.3) into (D.1), we obtain

$$f_{\gamma_i}^{(UB)}(x) = \sum_{\substack{k \in \{(s,i),(i,d)\} \\ j \neq k}} \left(\frac{1+K_k}{\Omega_k} e^{-K_k}\right) e^{-x\left(\frac{1+K_k}{\Omega_k}\right)} I_0\left(2\sqrt{\frac{K_k(1+K_k)x}{\Omega_k}}\right) Q\left(\sqrt{2K_j}, \sqrt{\frac{2(1+K_j)x}{\Omega_j}}\right) \tag{D.4}$$

Now the MGF of $\gamma_i^{(UB)}$ can be computed as

$$\phi_{\gamma_i}^{(UB)}(s) = \sum_{\substack{k \in \{(s,i),(i,d)\} \\ j \neq k}} \left(\frac{1+K_k}{\Omega_k} e^{-K_k}\right) \int_0^\infty e^{-x\left(s+\frac{1+K_k}{\Omega_k}\right)} I_0\left(2\sqrt{\frac{K_k(1+K_k)x}{\Omega_k}}\right) Q\left(\sqrt{2K_j}, \sqrt{\frac{2(1+K_j)x}{\Omega_j}}\right) dx \tag{D.5}$$

The above integral can be evaluated in closed-form using the identity [28, eq. (46)], which can be simplified into (6) after some standard algebraic manipulations. The MGF of lower bound for half harmonic mean SNR $\gamma_i^{(LB)} = \gamma_i^{(UB)}/2$ is computed as $\phi_{\gamma_i}^{(LB)}(s) = \phi_{\gamma_i}^{(UB)}(s/2)$.

APPENDIX E

In this appendix, we derive the MGF of upper bound for the half harmonic mean SNR of dual-hop CAF relayed path over an i.n.d Nakagami-m fading environment. In this case, the PDF and the CDF of fading SNR γ_q in (D.1) are given by [27, pp. 349], viz.,

$$f_{\gamma_q}(x) = \left(\frac{m_q}{\Omega_q}\right)^{m_q} \frac{1}{\Gamma(m_q)} x^{m_q-1} e^{-\frac{xm_q}{\Omega_q}}, \quad x \geq 0 \tag{E.1}$$

$$F_{\gamma_q}(x) = 1 - \frac{\Gamma\left(m_q, \frac{m_q x}{\Omega_q}\right)}{\Gamma(m_q)} \tag{E.2}$$

where m_q is the Nakagami-m fading severity index and $\Gamma(a,x) = \int_x^\infty t^{a-1} e^{-t} dt$ denotes the upper incomplete Gamma function.

Substituting (E.1) and (E.2) in to (D.1), we obtain

$$f_{\gamma_i}^{(UB)}(x) = \sum_{\substack{k \in \{(s,i),(i,d)\} \\ j \neq k}} \frac{1}{\Gamma(m_k)\Gamma(m_j)} \left(\frac{m_k}{\Omega_k}\right)^{m_k} x^{m_k - 1} e^{-\frac{x m_k}{\Omega_k}} \Gamma\left(m_j, \frac{m_j x}{\Omega_j}\right) \quad \text{(E.3)}$$

Taking the Laplace transform of (E.3), we get the MGF of $\gamma_i^{(UB)}$ as

$$\phi_{\gamma_i}^{(UB)}(s) = \sum_{\substack{k \in \{(s,i),(i,d)\} \\ j \neq k}} \frac{1}{\Gamma(m_k)\Gamma(m_j)} \left(\frac{m_k}{\Omega_k}\right)^{m_k} \int_0^\infty x^{m_k - 1} e^{-x\left(\frac{s\Omega_k + m_k}{\Omega_k}\right)} \Gamma\left(m_j, \frac{m_j x}{\Omega_j}\right) dx \quad \text{(E.4)}$$

The above integral can be evaluated in closed-form using [20, eq. (6.455.1)] and Kummer's transformation identity [20, eq. (9.131.1)], viz.,

$$\int_0^\infty x^{\mu-1} e^{-\beta x} \Gamma(v, \alpha x) dx = \frac{\Gamma(\mu + v)}{\mu(\alpha + \beta)^\mu} {}_2F_1\left(\mu, 1 - v; \mu + 1; \frac{\beta}{\alpha + \beta}\right) \quad \text{(E.5)}$$

where ${}_2F_1(a, b; c; z)$ denotes the Gauss hypergeometric function.

Next, simplifying (E.4) using identity (E.5), we obtain the MGF of $\gamma_i^{(UB)}$ as shown in (8).

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